

# Cryptanalysis of Alternative Hash Functions

Florian Mendel

<http://www.iaik.tugraz.at/aboutus/people/mendel/index.php>

Hash&Stream, Salzburg, 2007/02/02

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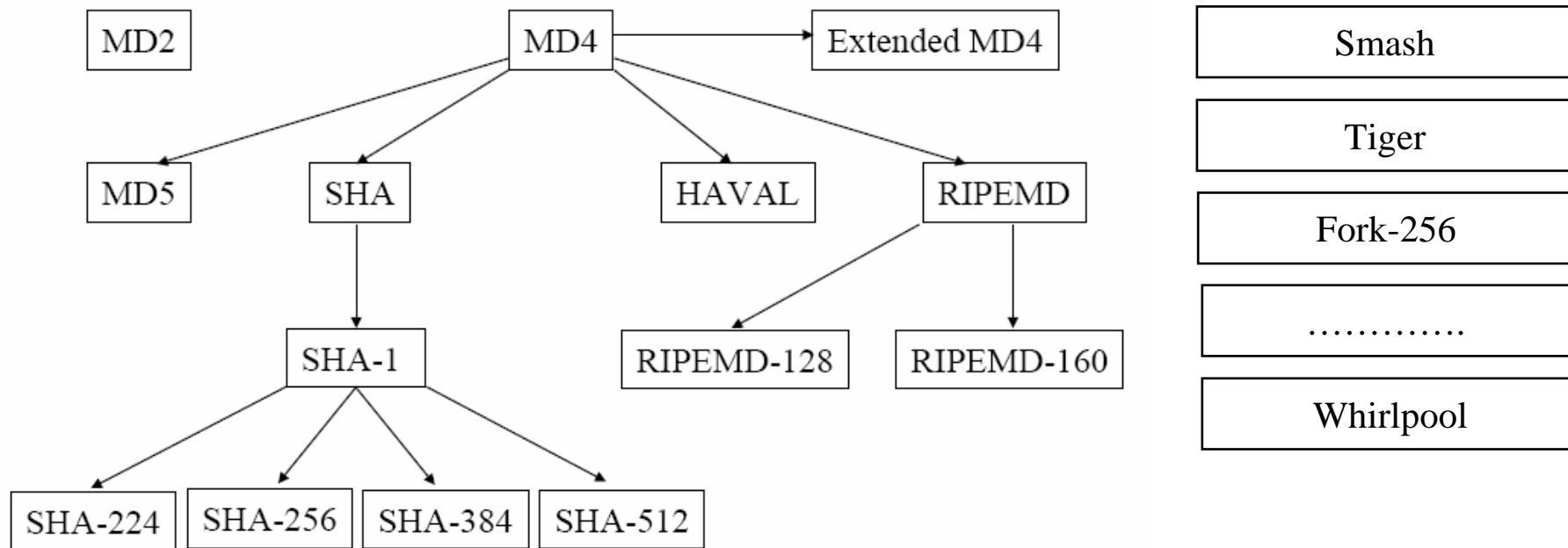
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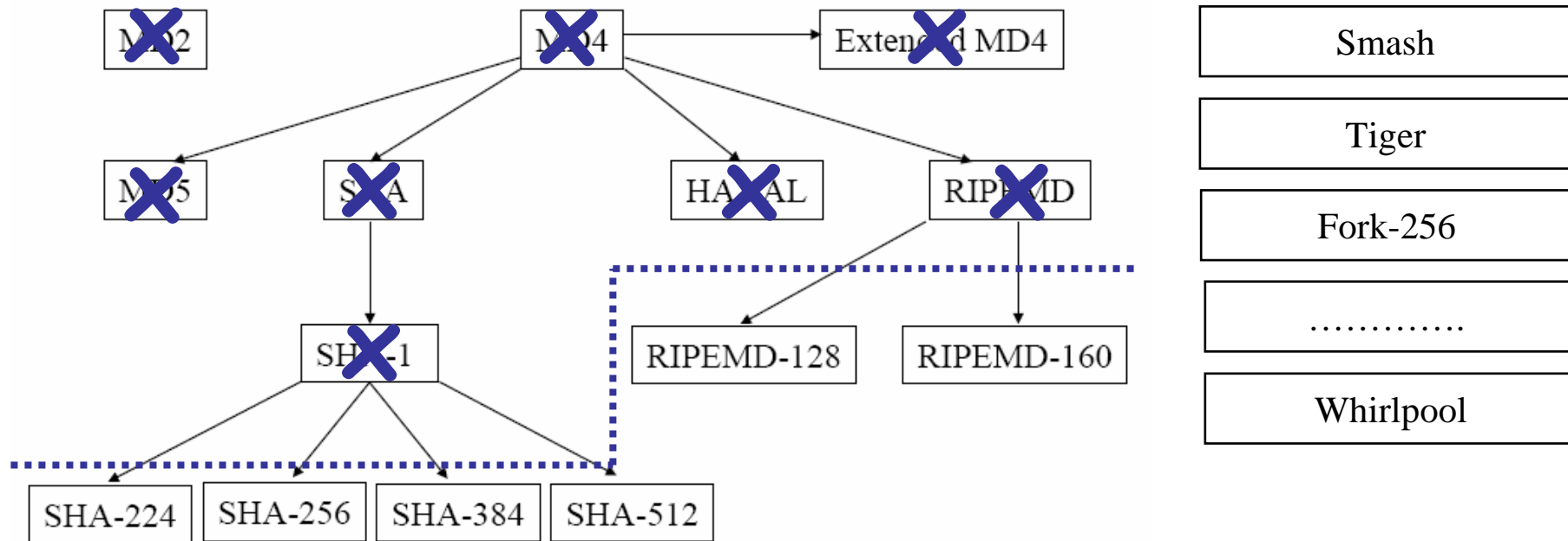
# Outline

- Motivation
- Cryptanalysis of
  - SHA-256
  - RIPEMD-160 and RIPEMD-128
  - Smash
  - Tiger
- Conclusion

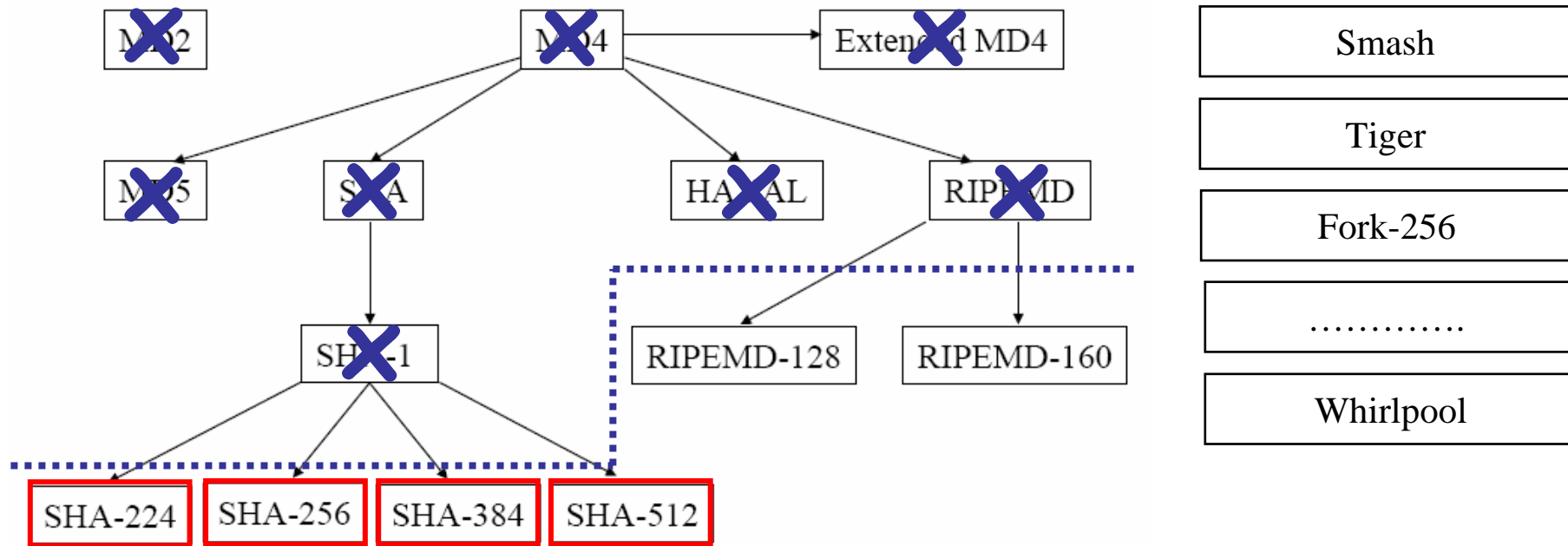
# Motivation



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# Analysis of Step-Reduced SHA-256

Florian Mendel and Norbert Pramstaller and  
Christian Rechberger and Vincent Rijmen

**presented at FSE 2006**

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# SHA-256 is Interesting and Challenging

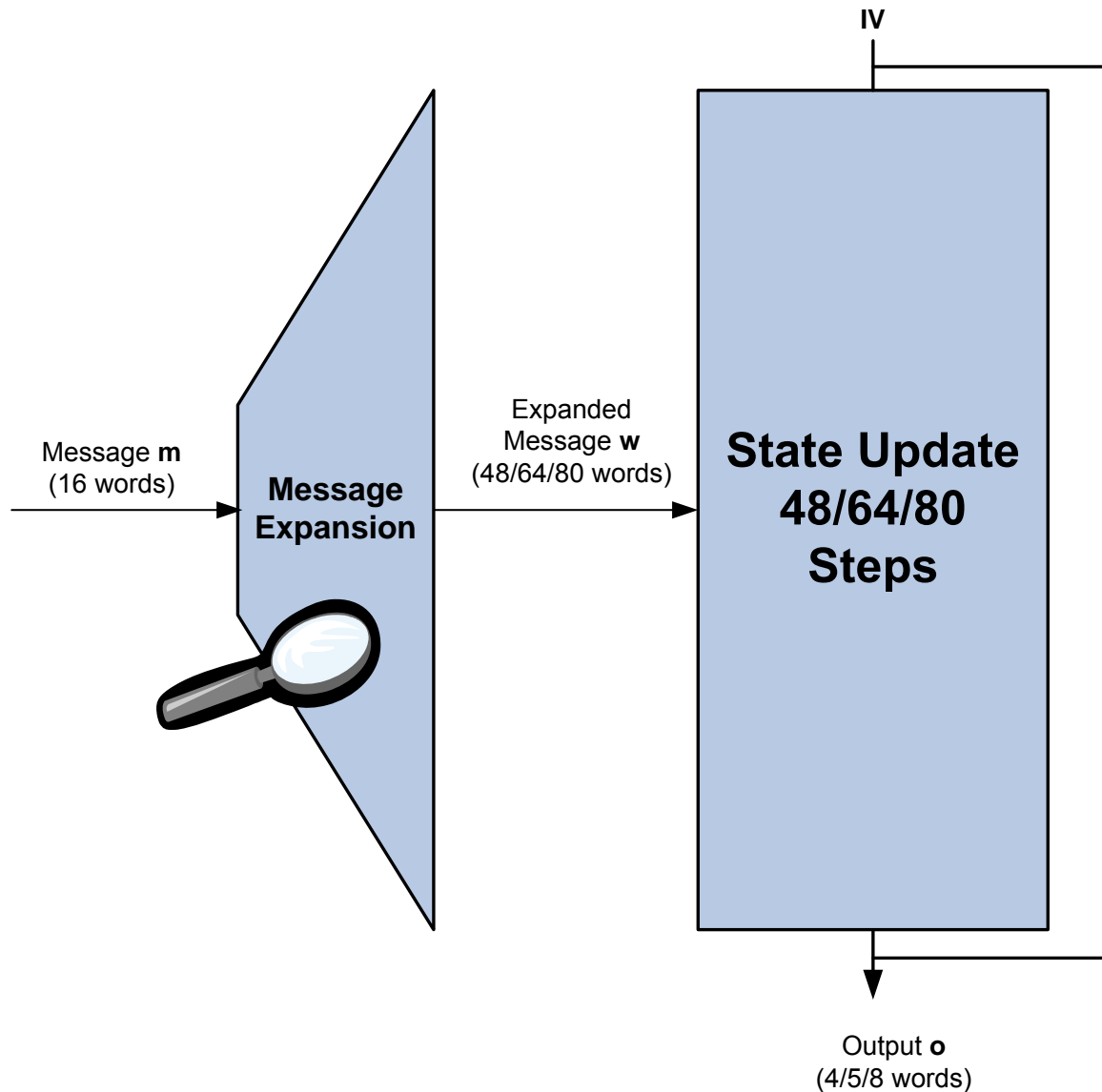
FIPS Standard since 2002  
Option for a SHA-1 upgrade



Prudent to know:

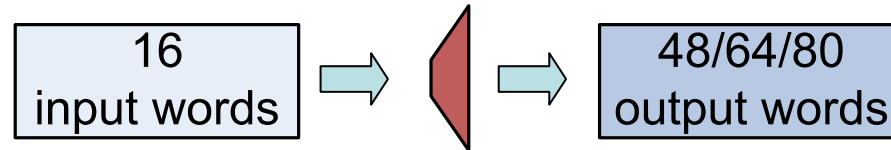
How hard is it to find collisions for SHA-256?  
What about step-reduced variants (security margin)?

# Outline of MD4-style Hash Functions

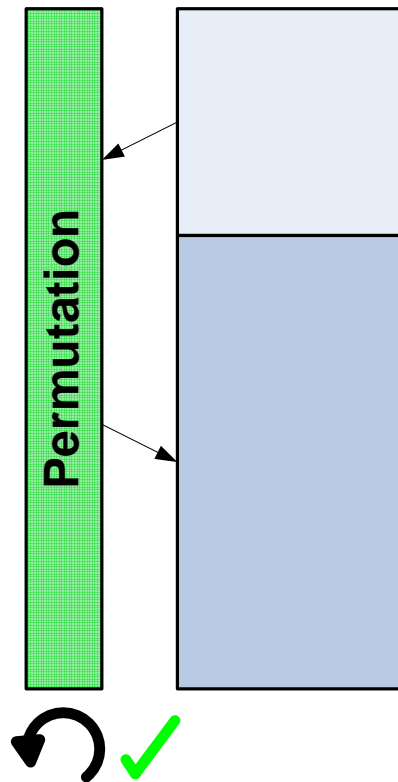




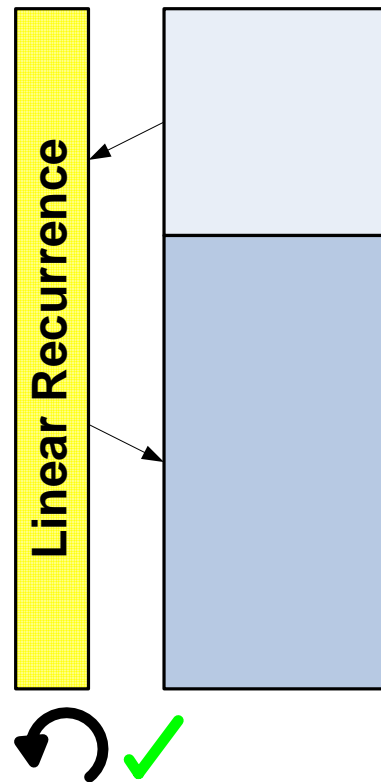
# Message Expansions in the MD4 family



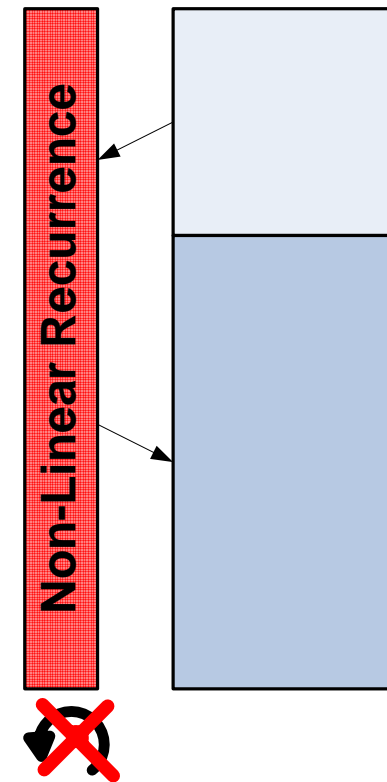
**MD4/5, RIPEMD**



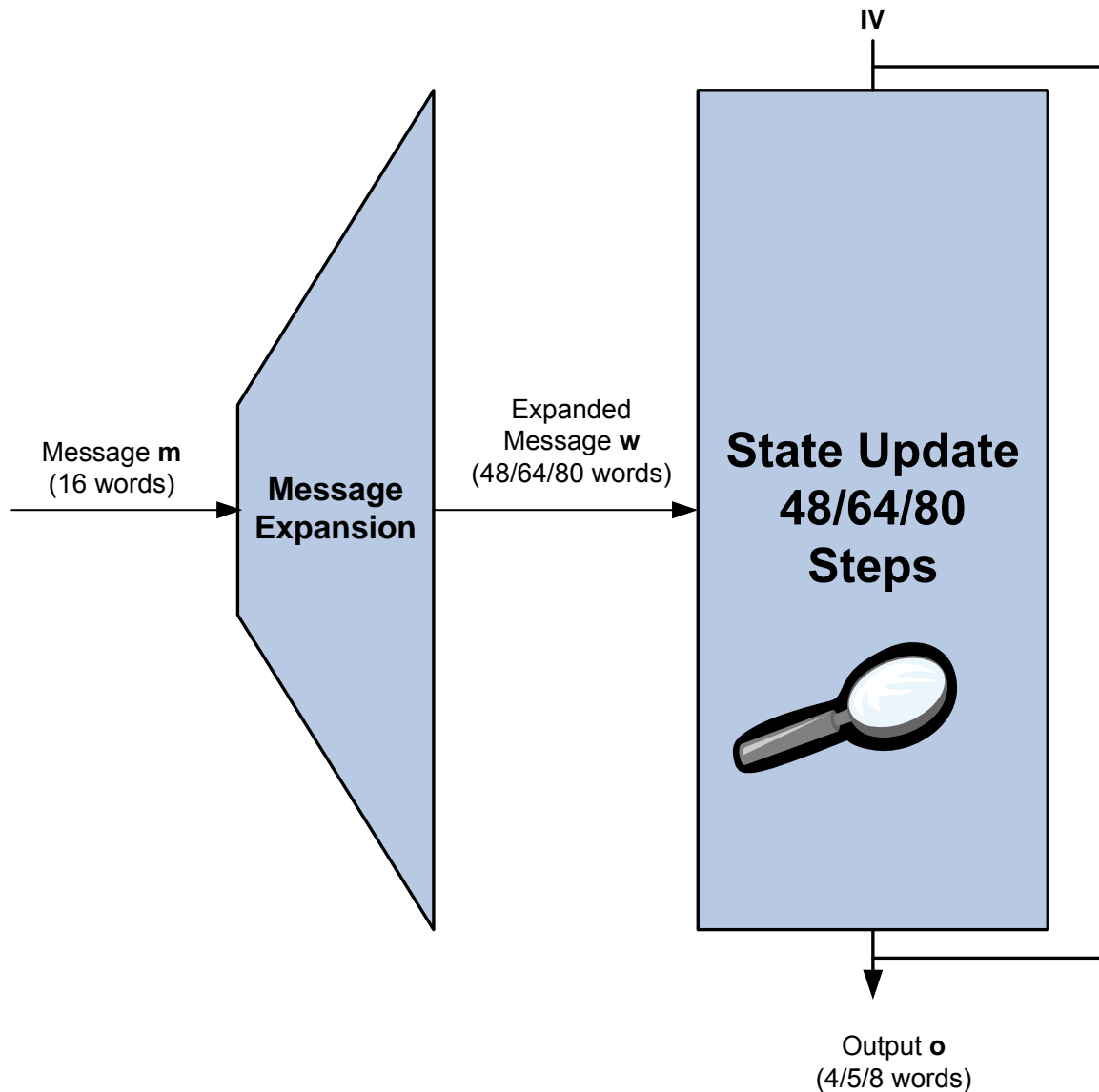
**SHA-0 / SHA-1**



**SHA-2 family**

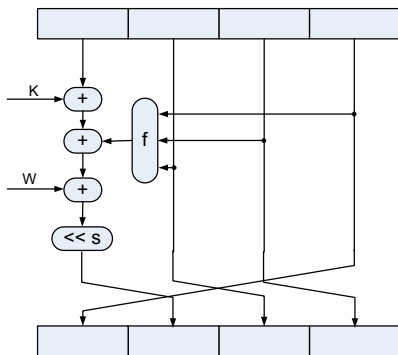


# Outline of MD4-style Hash Functions

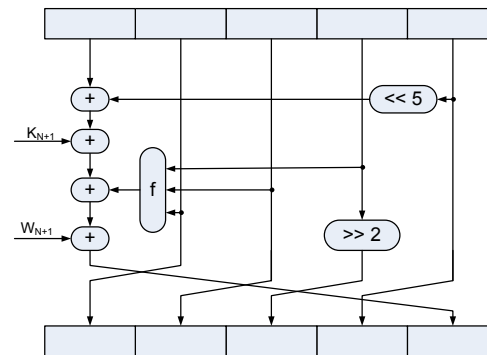


# Evolution of the State Updates in the MD4 Family

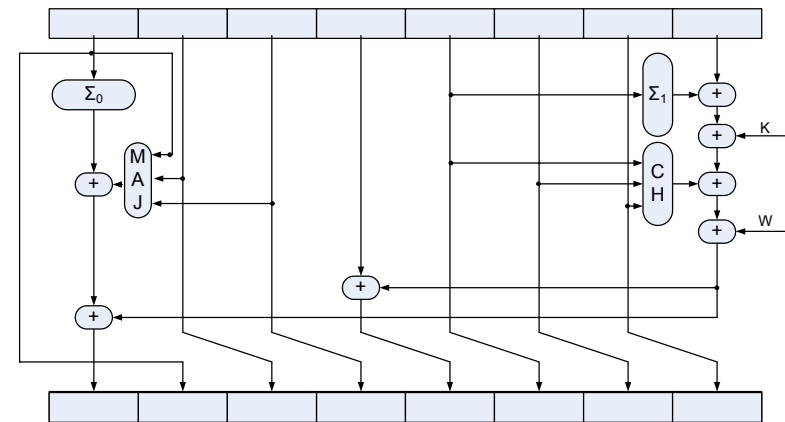
MD4



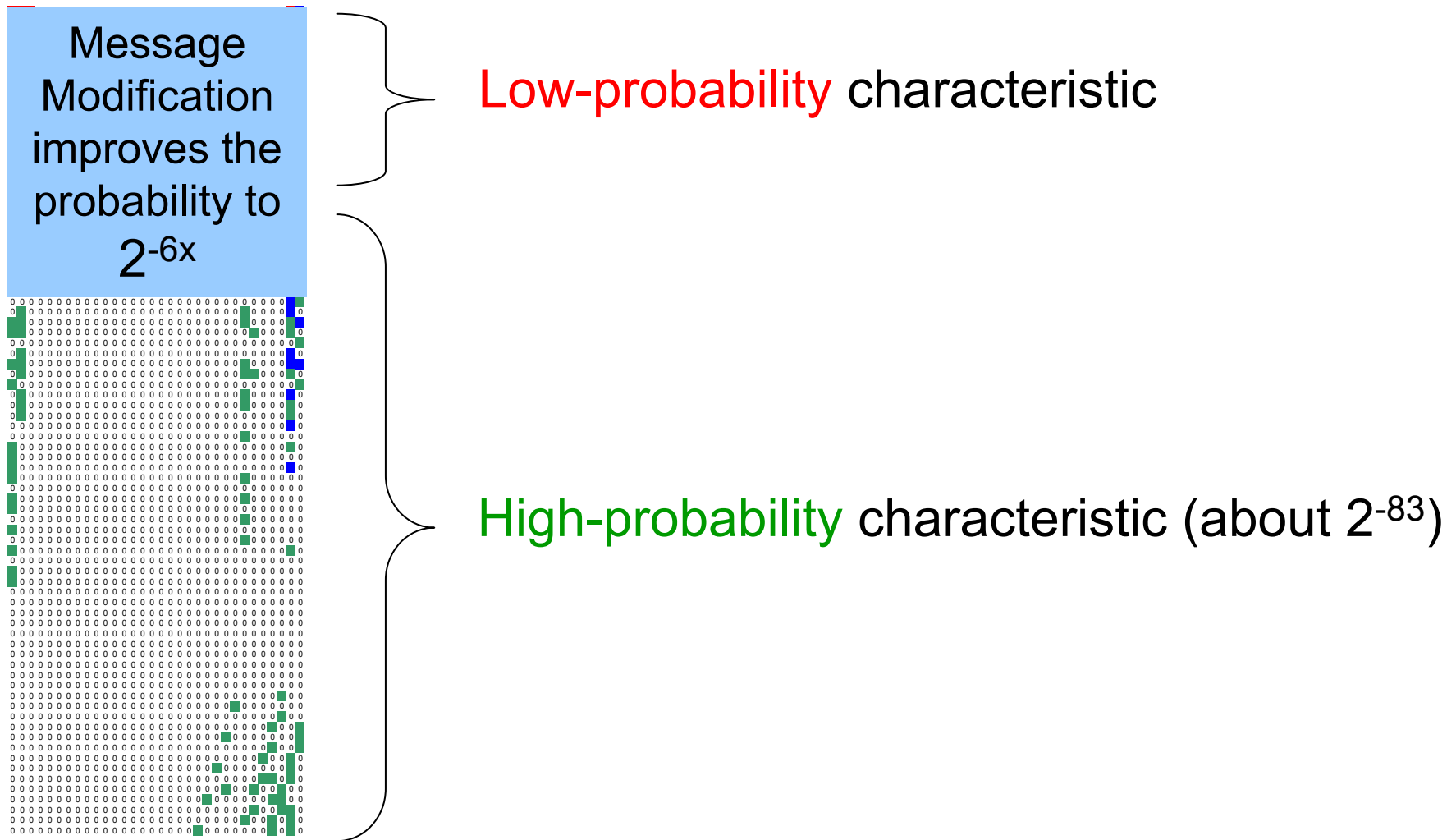
SHA-0/1



SHA-2 family

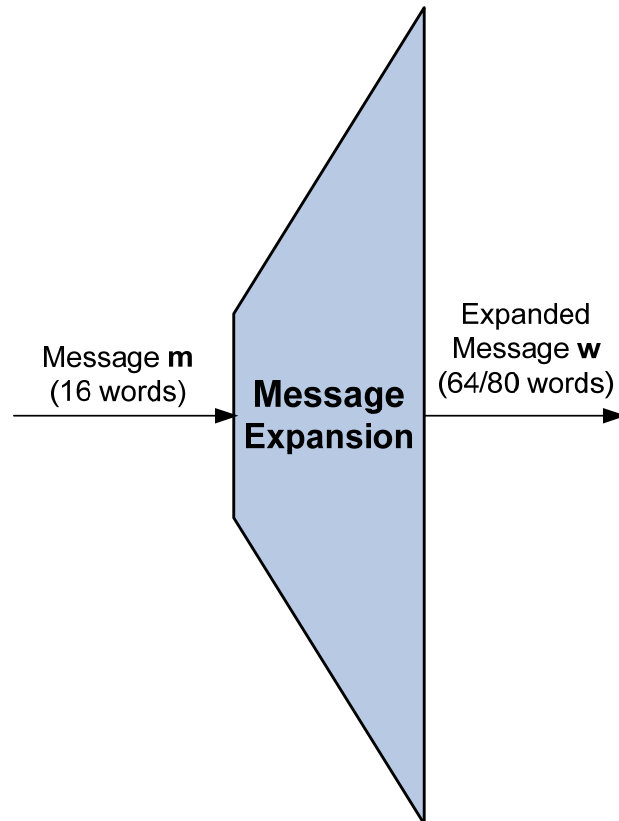


# Attack of Wang *etal.* On SHA-1



# Comparison of SHA Message Expansions

## SHA-1



$$W_t = \begin{cases} M_t & \text{for } (0 \leq t \leq 15) \\ \text{ROTL}^1(W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16}) & \text{for } (16 \leq t \leq 79) \end{cases}$$

## SHA-256

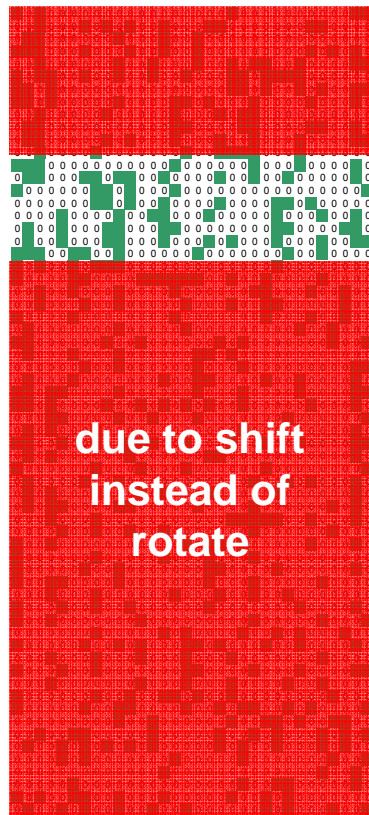
$$W_t = \begin{cases} M_t & \text{for } (0 \leq t \leq 15) \\ \sigma_1(W_{t-2}) + W_{t-7} + \sigma_0(W_{t-15}) + W_{t-16} & \text{for } (16 \leq t \leq 63) \end{cases}$$

$$\sigma_0(x) = \text{ROTR}^7(x) \oplus \text{ROTR}^{18}(x) \oplus \text{SHR}^3(x)$$

$$\sigma_1(x) = \text{ROTR}^{17}(x) \oplus \text{ROTR}^{19}(x) \oplus \text{SHR}^{10}(x)$$



# Approach does not apply to SHA-2



Low-probability characteristic

~~High-probability characteristic~~  
Low-probability characteristic

## Example of 19-step Characteristic

Step	W'	A'	B'	C'	D'	E'	F'	G'	H'
1-4	0	0	0	0	0	0	0	0	0
05	85009008	85009008	0	0	0	85009008	0	0	0
06	a14cae12	a1442610	85009008	0	0	02000802	85009008	0	0
07	0	0	a1442610	85009008	0	084c4120	02000802	85009008	0
08	8200a8a8	00000020	0	a1442610	85009008	00000020	084c4120	02000802	85009008
09	85009008	85009008	00000020	0	a1442610	01008008	00000020	084c4120	02000802
10	0	0	85009008	00000020	0	02000802	01008008	00000020	084c4120
11	0	0	0	85009008	00000020	0	02000802	01008008	00000020
12	0	00000020	0	0	85009008	0	0	02000802	01008008
13	0	0	00000020	0	0	84001000	0	0	02000802
14	00088802	0	0	00000020	0	0	84001000	0	0
15	0	0	0	0	00000020	0	0	84001000	0
16	0	0	0	0	0	00000020	0	0	84001000
17	0	0	0	0	0	0	00000020	0	0
18	0	0	0	0	0	0	0	00000020	0
19	0	0	0	0	0	0	0	0	00000020

**collision for SHA-224**

## Interesting Results

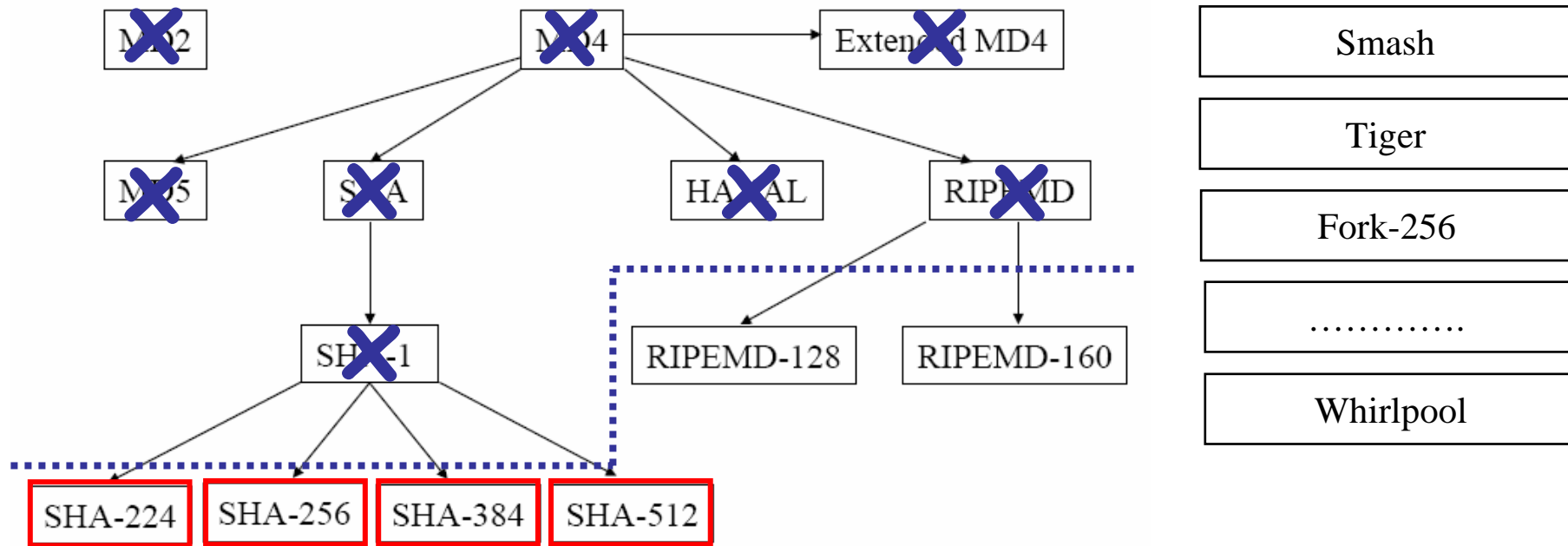
- Perturbation pattern is no valid expanded message
  - But the sum of perturbations and corrections is
  
- More freedom for the carry
  - ... to prevent contradictions in characteristics
  
- The overall probability is much higher than the product of the probabilities of each individual local collision
  - Different to SHA-0 / SHA-1
  - Example: low-weight 19-step characteristic
    - 23 local collisions of probability around  $2^{-40}$
    - Total probability is much higher: instead of  $2^{-920}$  around  $2^{-200}$   
(Compare this to a similar probability of the best known 80-step characteristic for SHA-1)



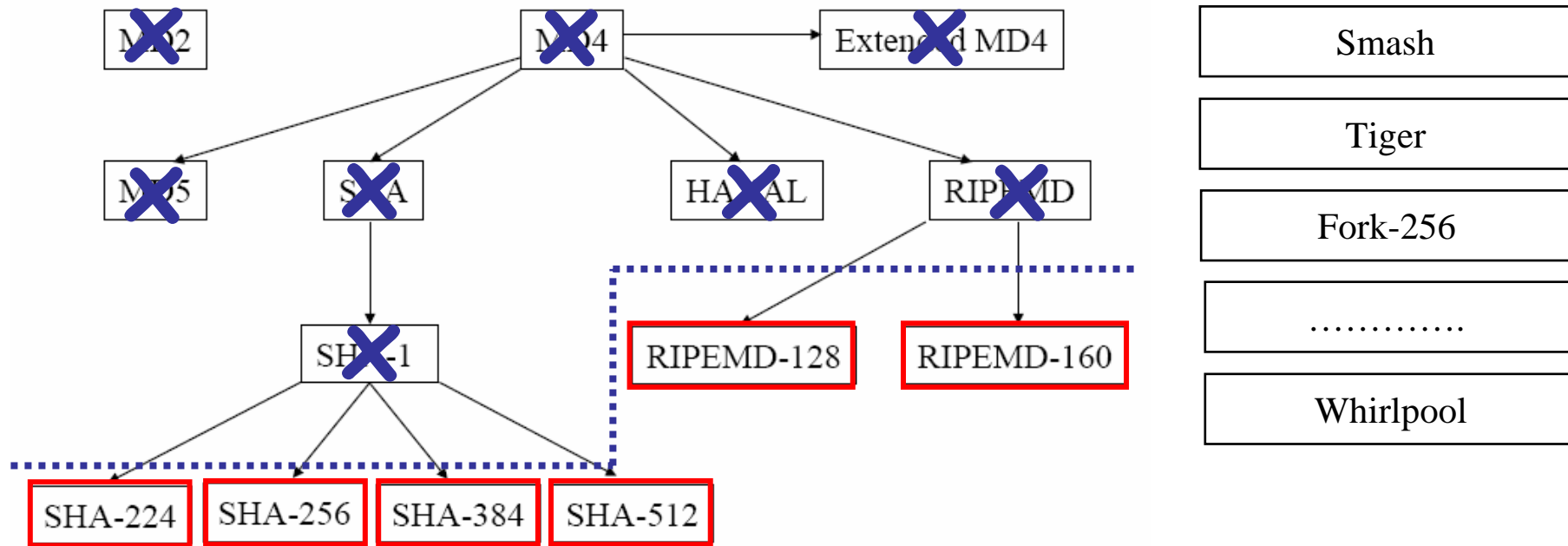
## Summary

- First analysis of unmodified SHA-256/224 for a nontrivial number of steps
  
- Collision resistance of SHA-256/224 is not threatened
- All publicly known attacks on SHA-0/1 since 1997 are not directly applicable to any SHA-2 member
  
- New analysis method
  - New type of perturbation pattern
  - Probability of a local collision is much less relevant
  - Explicit control of carry extensions is possible and needed

# Motivation



# Motivation



# On the Collision-Resistance of RIPEMD-160

Florian Mendel, Christian Rechberger,  
Norbert Pramstaller, and Vincent Rijmen

**presented at ISC 2006**

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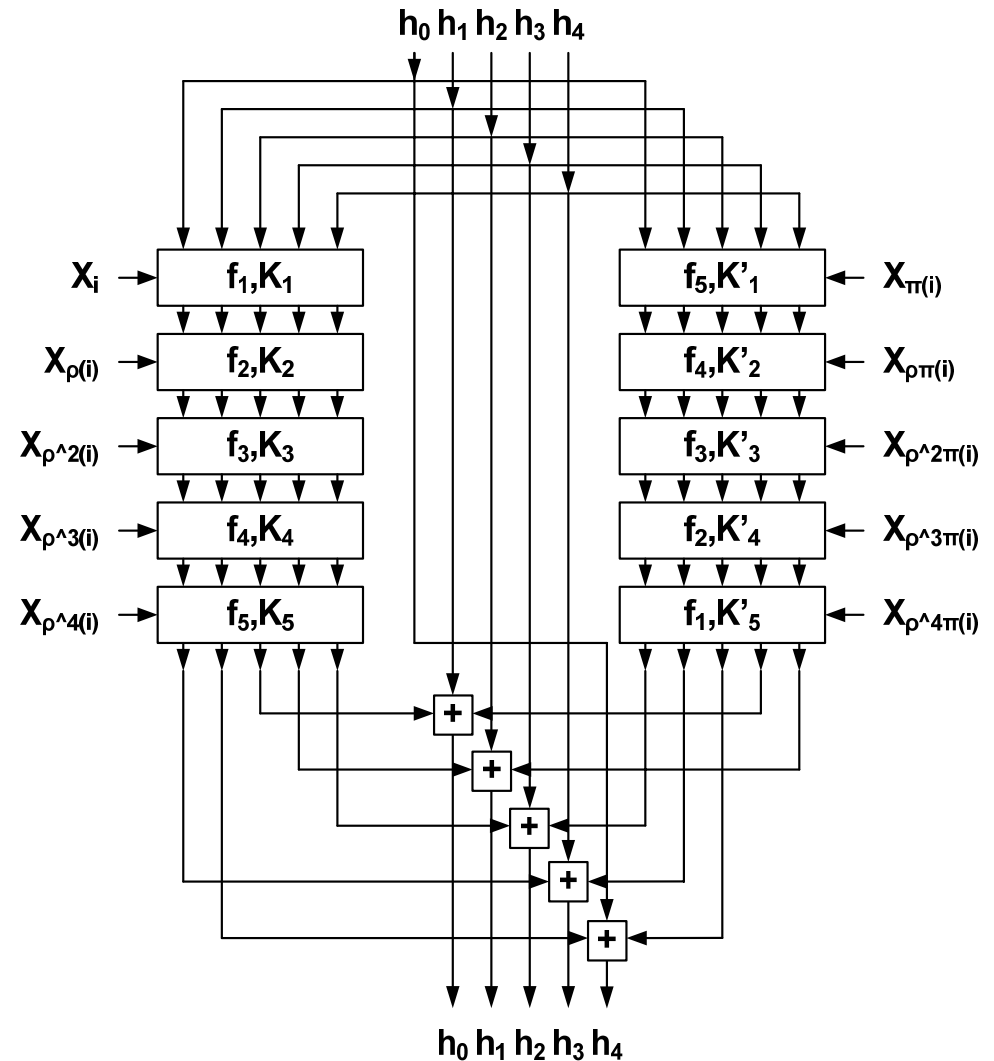


# The RIPEMD-family

- RIPEMD
  - Results by Dobbertin (round reduced)
  - Collisions announced in 2004 by Wang et al.
  
- Introduction of two strengthened versions
  - RIPEMD-128
  - RIPEMD-160
  
- RIPEMD-160 is frequently recommended
  
- Attacks extendable to RIPEMD-160?

# RIPEMD-160 / 128

- RIPEMD-160
  - Output is 160 bits
  - Process message in 16 words (512-bit)
  - Uses 10 rounds of 16 steps in **2 parallel lines of 5**
  
- RIPEMD-128
  - Output 128 bits
  - Uses 8 rounds of 16 steps in **2 parallel lines of 4**

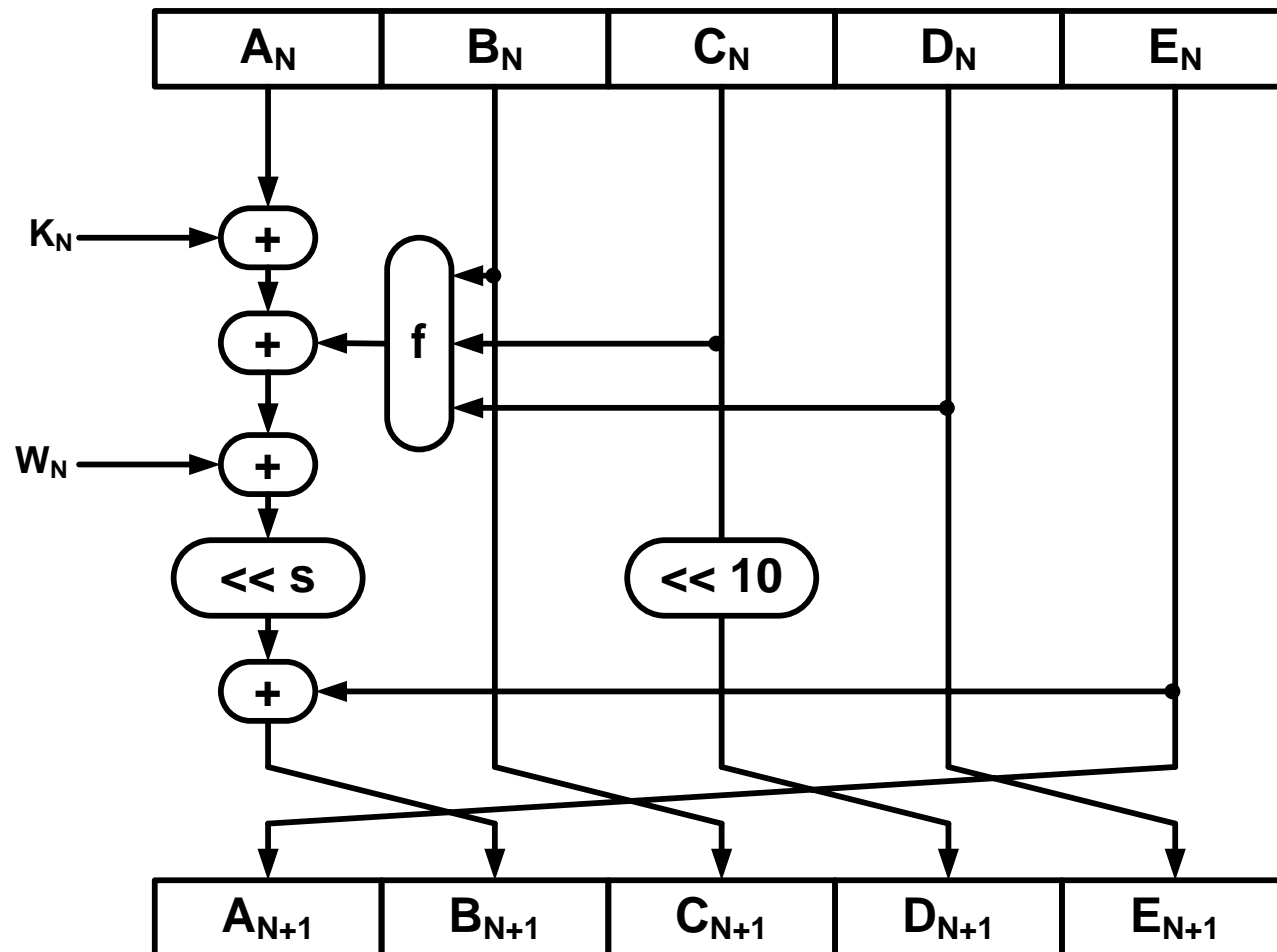


# Step Function of RIPEMD-160

Modular Additions

Boolean Functions

Variable Rotation



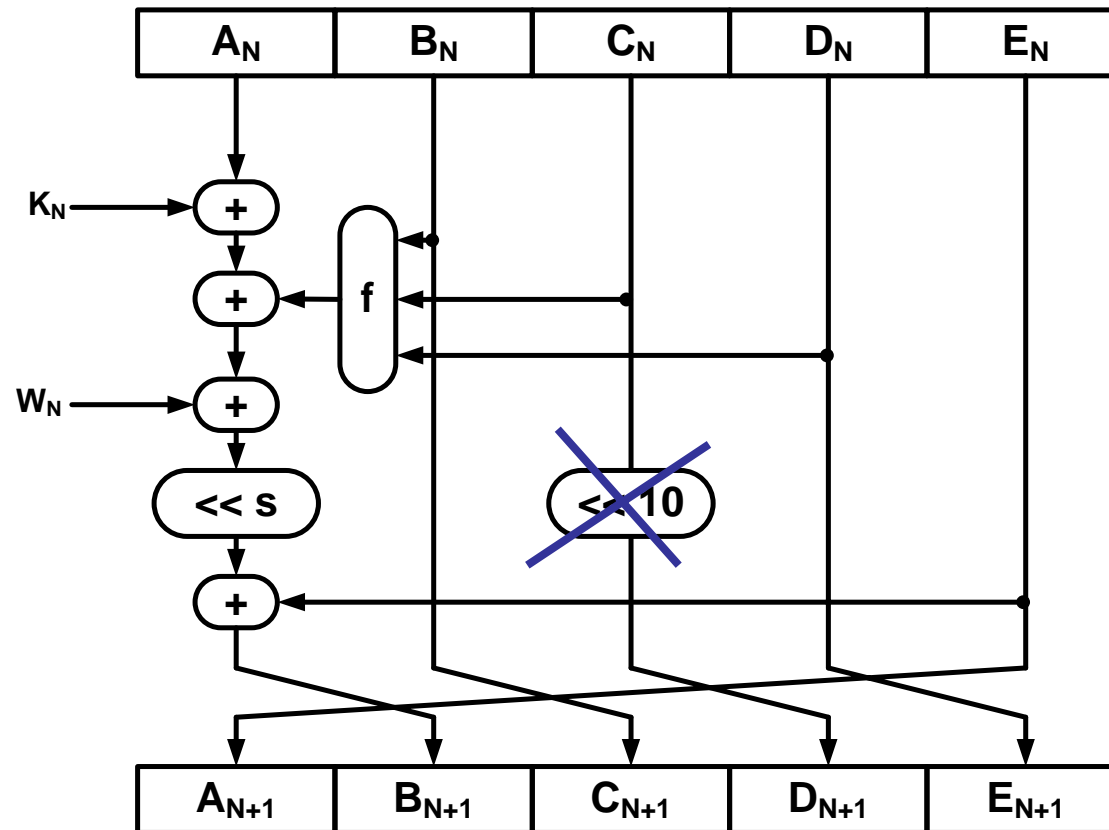
## Results of the low-weight search using a general characteristic

The attack of Wang *et al.* on SHA-1 does not apply to RIPEMD-160 – no characteristic with low Hamming weight can be found

	Hamming weight	Stream	#Steps
RIPEMD - 160	480	Both	17 - 80
	352	Both	17 - 64
	224	Both	17 - 48
RIPEMD - 128	448	Both	17 - 64
	18	Both	17 - 48



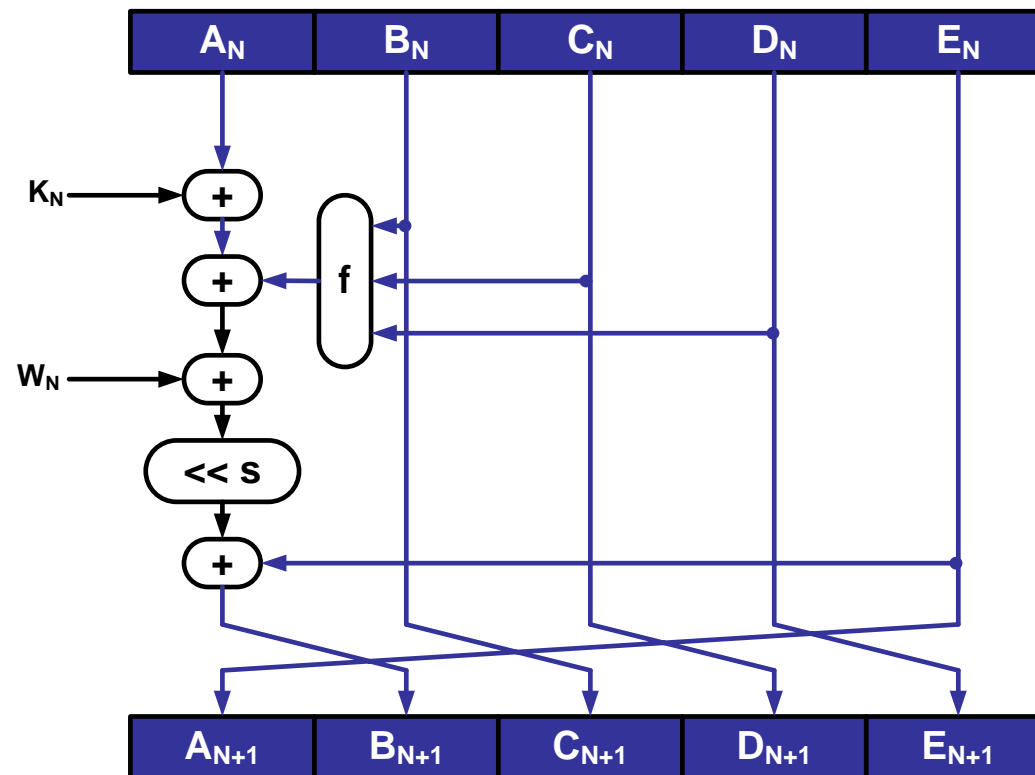
# A simplified variant of RIPEMD-160



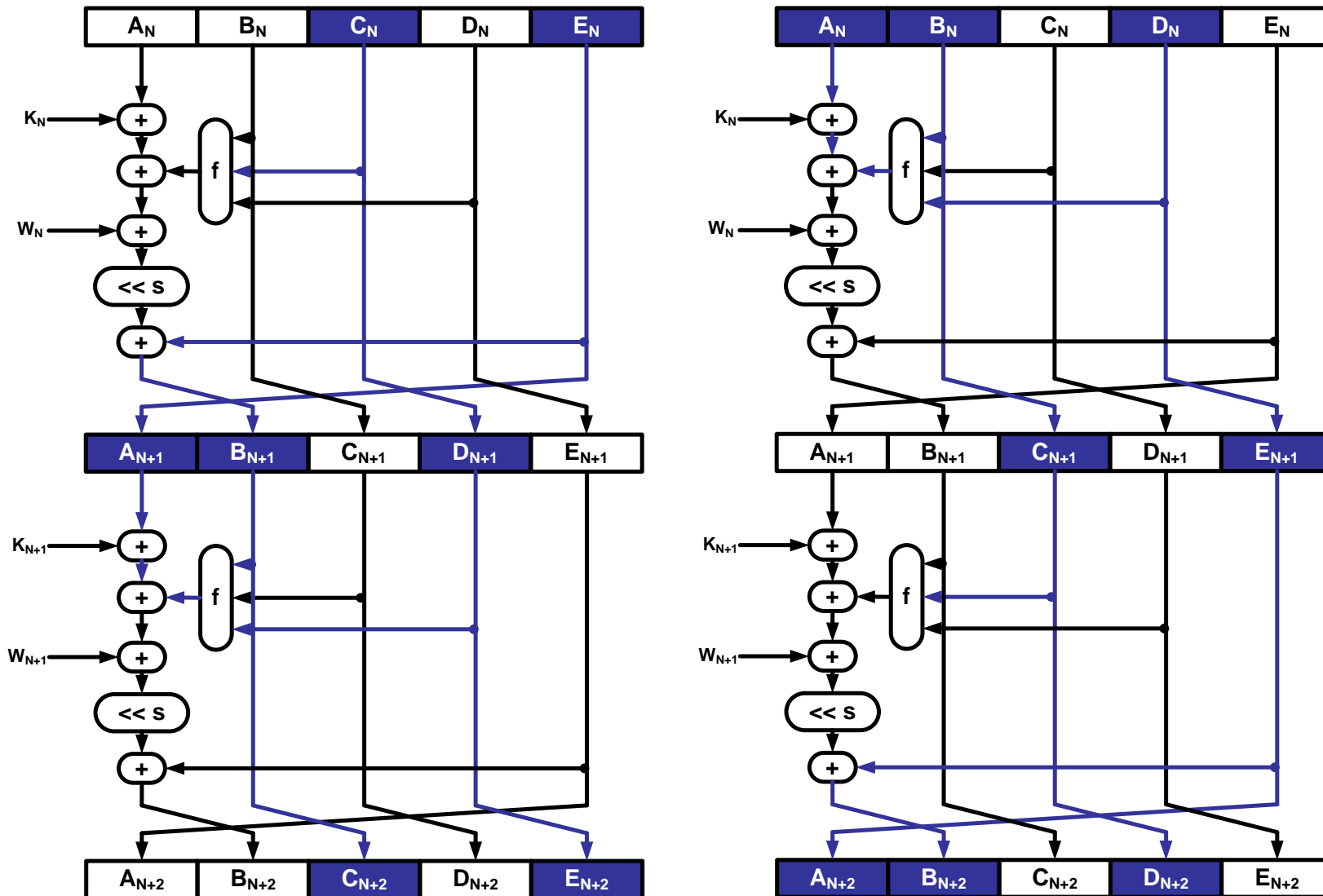
**Note:** Rotation of C is removed

# Fixed Points in the simplified variant of RIPEMD-160

- Input differences = output differences
- Properties of  $f$  can be used to cancel differences in  $W_T$



# Fixed Points (for 2 steps)



## Using fixed points for collisions

- Collision for RIPEMD-160 variant reduced to 3 rounds using fixed point  $FP_1$ :
  - 1 message block
  - 64 equations on  $A_N$
- Collision for RIPEMD-160 variant reduced to 3 rounds using fixed point  $FP_{2a}$  or  $FP_{2b}$ 
  - 5 message blocks
  - For each message block there are 48 equations on  $A_N$
- Theoretical attack for RIPEMD-160 variant reduced to 3 rounds

# Summary

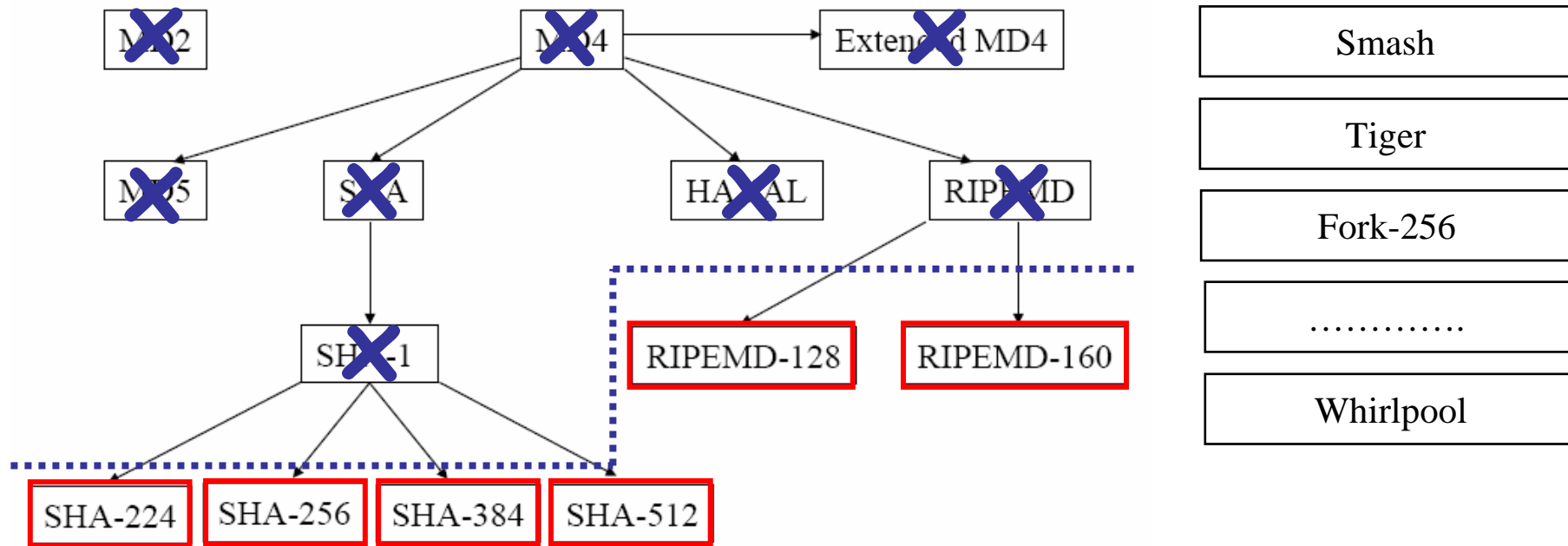
- Theoretical attack on 3 rounds of a simplified variant of RIPEMD-160
- So far no results for the original RIPEMD-160 hash function
  - Number of equations is too large
  - No differential pattern found with low Hamming weight
- RIPEMD-160 seems to be secure against these kind of collision-attacks

# Summary

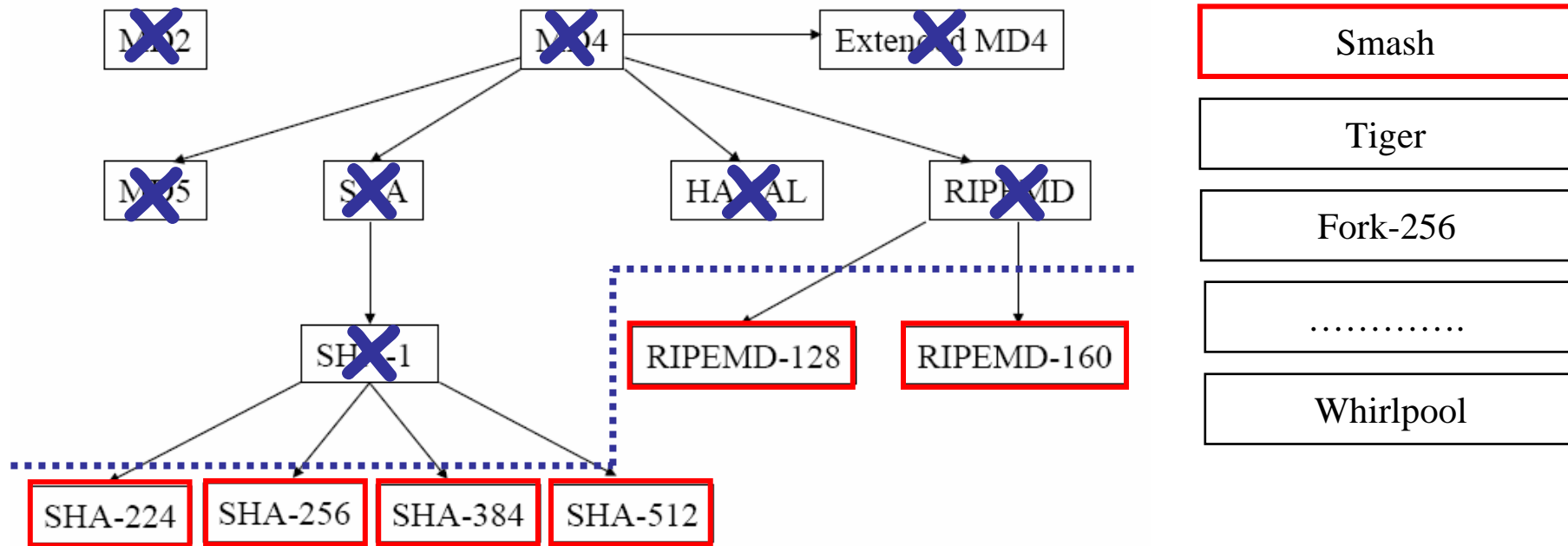
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  - Number of equations is too large
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**Further analysis is required to get a good view on the security margins of RIPEMD-160 and RIPEMD-128**

# Motivation



# Motivation





# Structural Analysis of SMASH

Mario Lamberger, Norbert Pramstaller,  
Christian Rechberger, and Vincent Rijmen

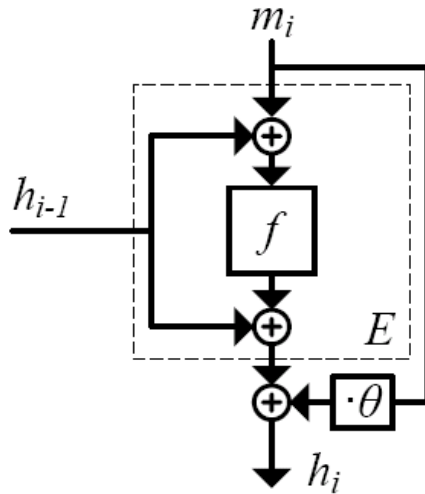
**presented at SAC 2005 and CT-RSA 2007**

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# SMASH Design Strategy



$$h_0 = f(iv) + iv$$

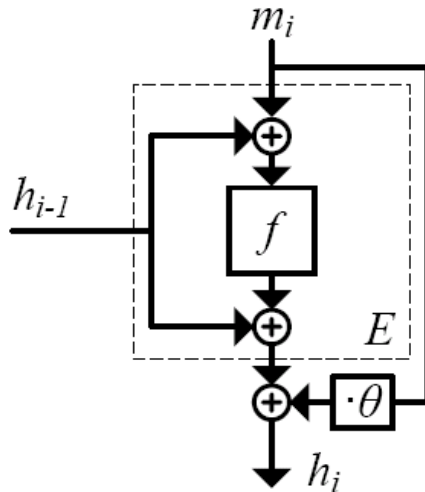
$$h_i = f(h_{i-1} + m_i) + h_{i-1} + \theta m_i \quad \text{for } i = 1, \dots, t$$

$$h_{t+1} = f(h_t) + h_t .$$

- Compression function based on nonlinear bijective  $n$ -bit mapping  $f$
- $\theta$  is an arbitrary field element in  $GF(2^n)$  with  $\theta \neq \{0, 1\}$

+ ... addition in  $GF(2^n)$

# SMASH Design Strategy



$$h_0 = f(iv) + iv$$

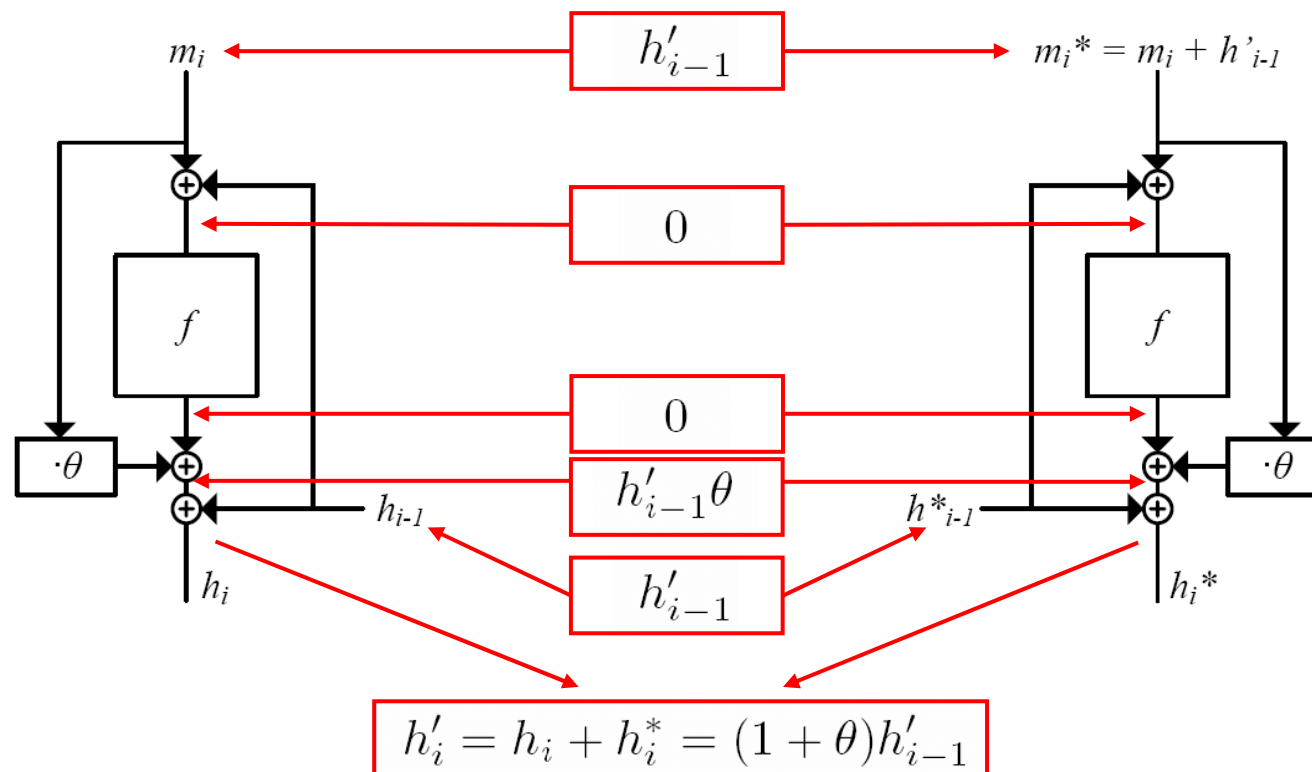
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- Compression function based on nonlinear bijective n-bit mapping  $f$
  - $\theta$  is an arbitrary field element in  $GF(2^n)$  with  $\theta \neq \{0, 1\}$
  - Specific instance: SMASH-256 ( $n=256$ )
    - $GF(2^{256})$  defined by  $q(\alpha) = \alpha^{256} + \alpha^{16} + \alpha^3 + \alpha + 1$
    - Element  $\theta$  is defined as root of  $q(\alpha)$
- + ... addition in  $GF(2^n)$

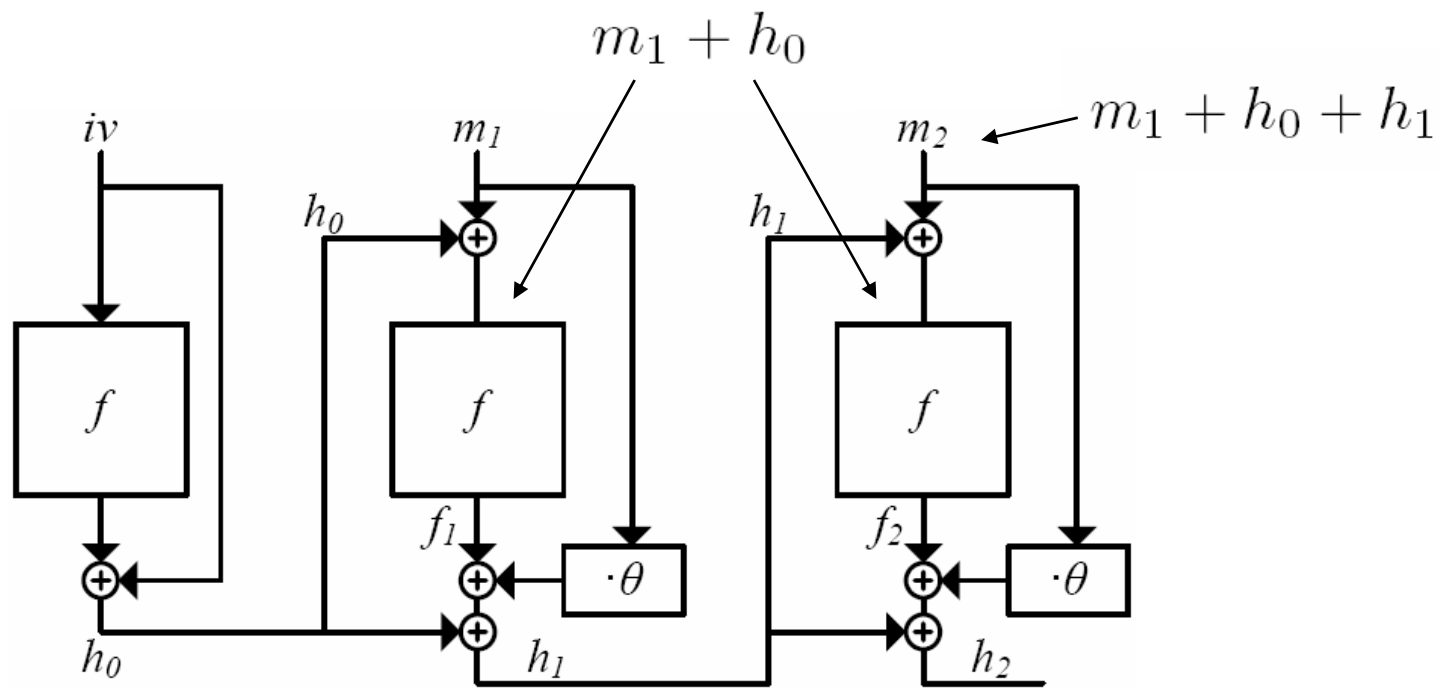
# Forward Prediction Property (FPP)

- Given intermediate hash values  $h_{i-1}, h_{i-1}^*$  with difference  $h'_{i-1} = h_{i-1} + h_{i-1}^*$
- Choose  $m_i$  and compute  $m_i^* = m_i + h'_{i-1}$



# Pattern Construction Property (PCP)

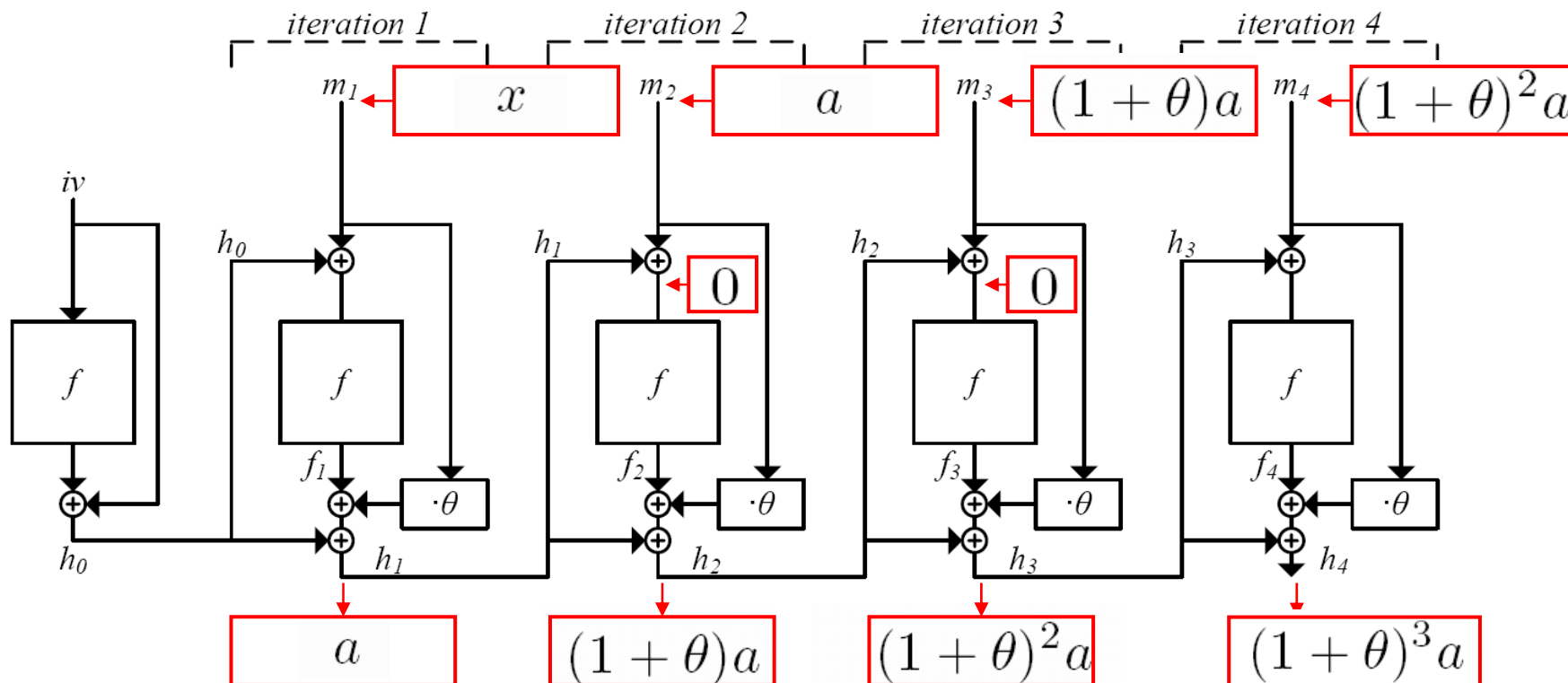
- Input of  $f$  must be the same for both iterations



$$m_2 = m_1 + f_1 + \theta m_1 \Rightarrow f_2 = f_1$$

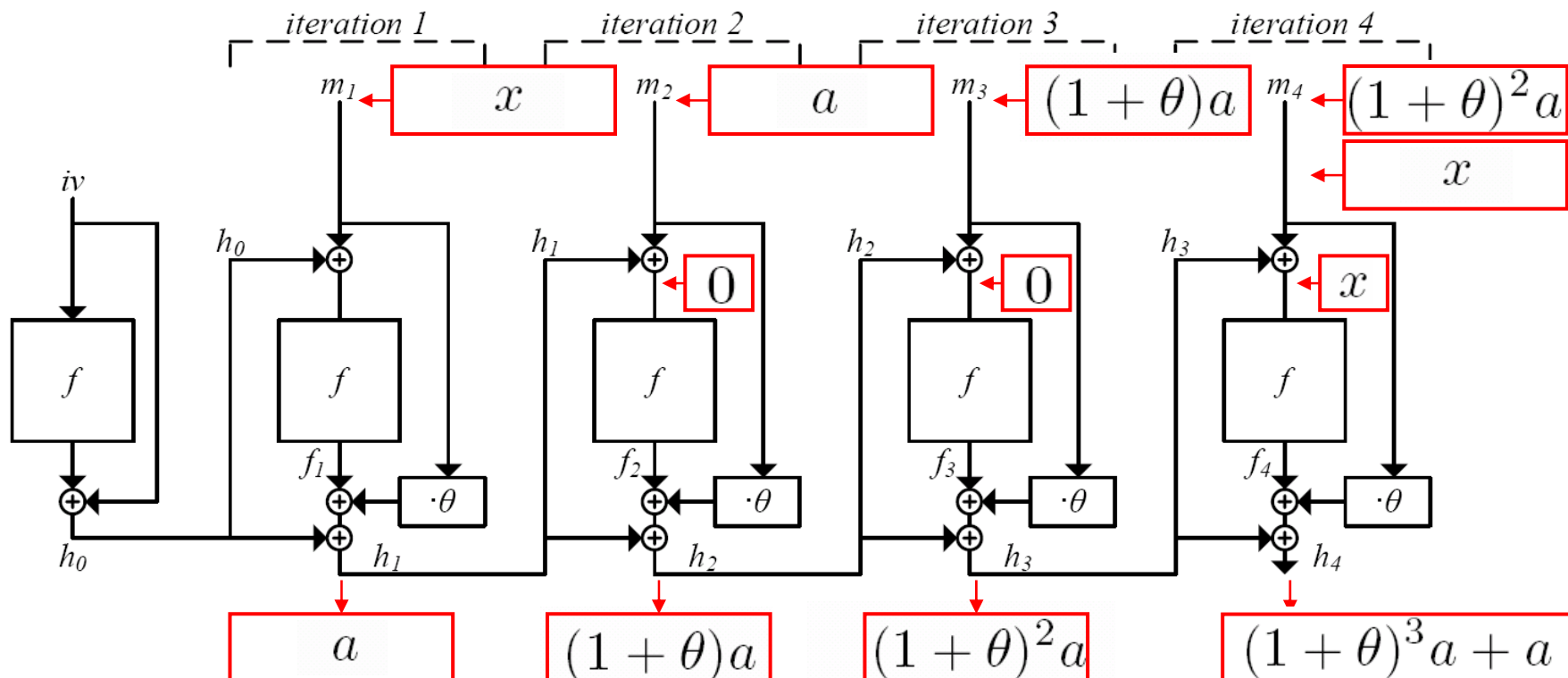
# Exploiting FPP/PCP for Collisions – The Principle

- Assume we can choose a  $\theta$  such that  $(1 + \theta)^3 = 1$



# Exploiting FPP/PCP for Collisions – The Principle

- Assume we can choose a  $\theta$  such that  $(1 + \theta)^3 = 1$

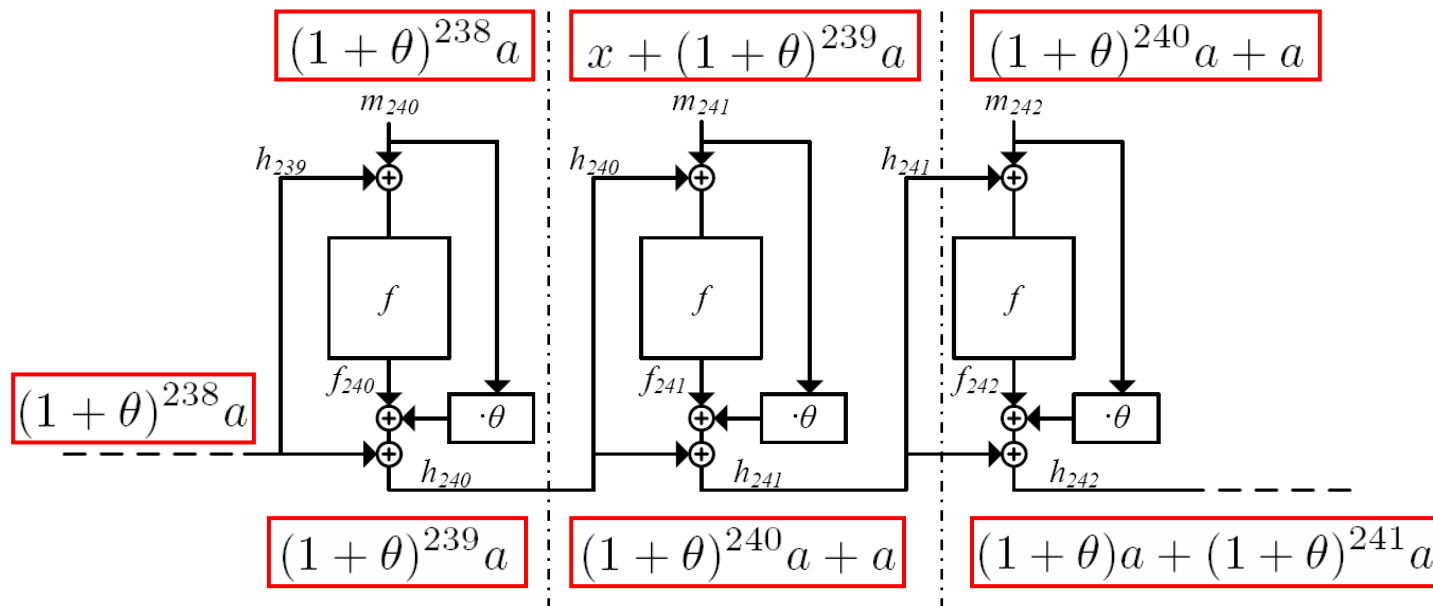


# Exploiting FPP/PCP for Collisions in SMASH-256

- For a collision we need

$$a \cdot q(\theta) = (1 + \theta)^{256} a + (1 + \theta)^{16} a + (1 + \theta)^3 a + (1 + \theta)^2 a + a$$

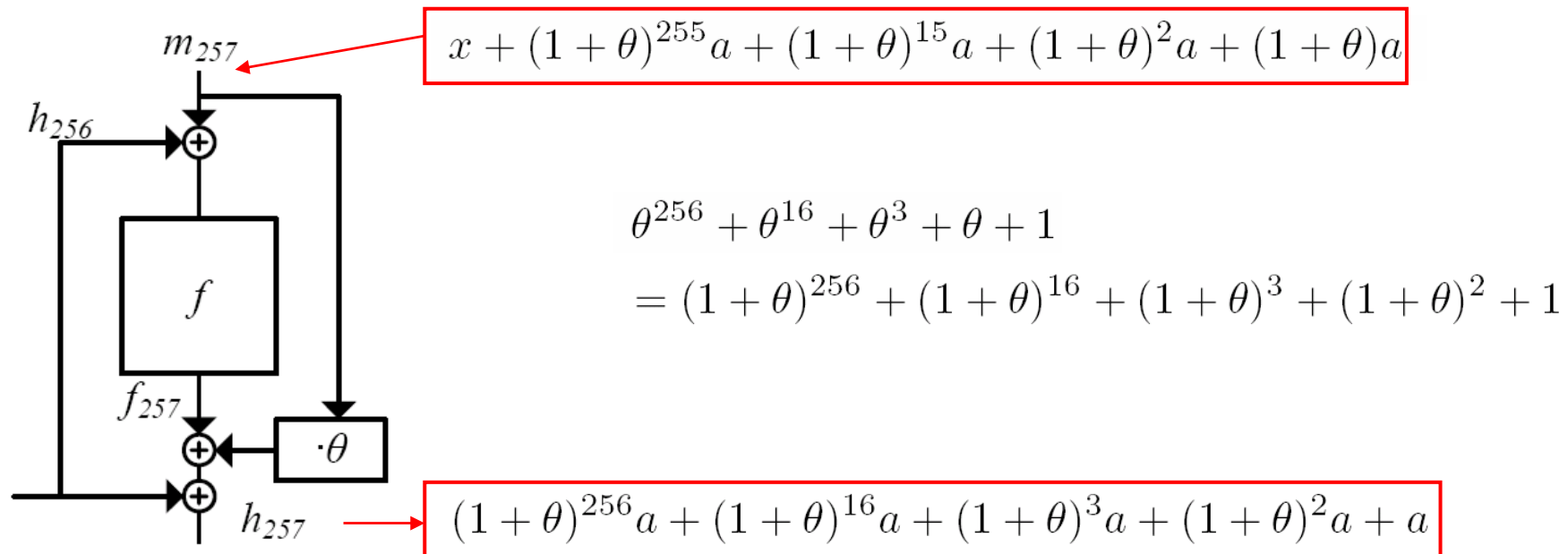
- Constructing the polynomial





# Exploiting FPP/PCP for Collisions in SMASH-256

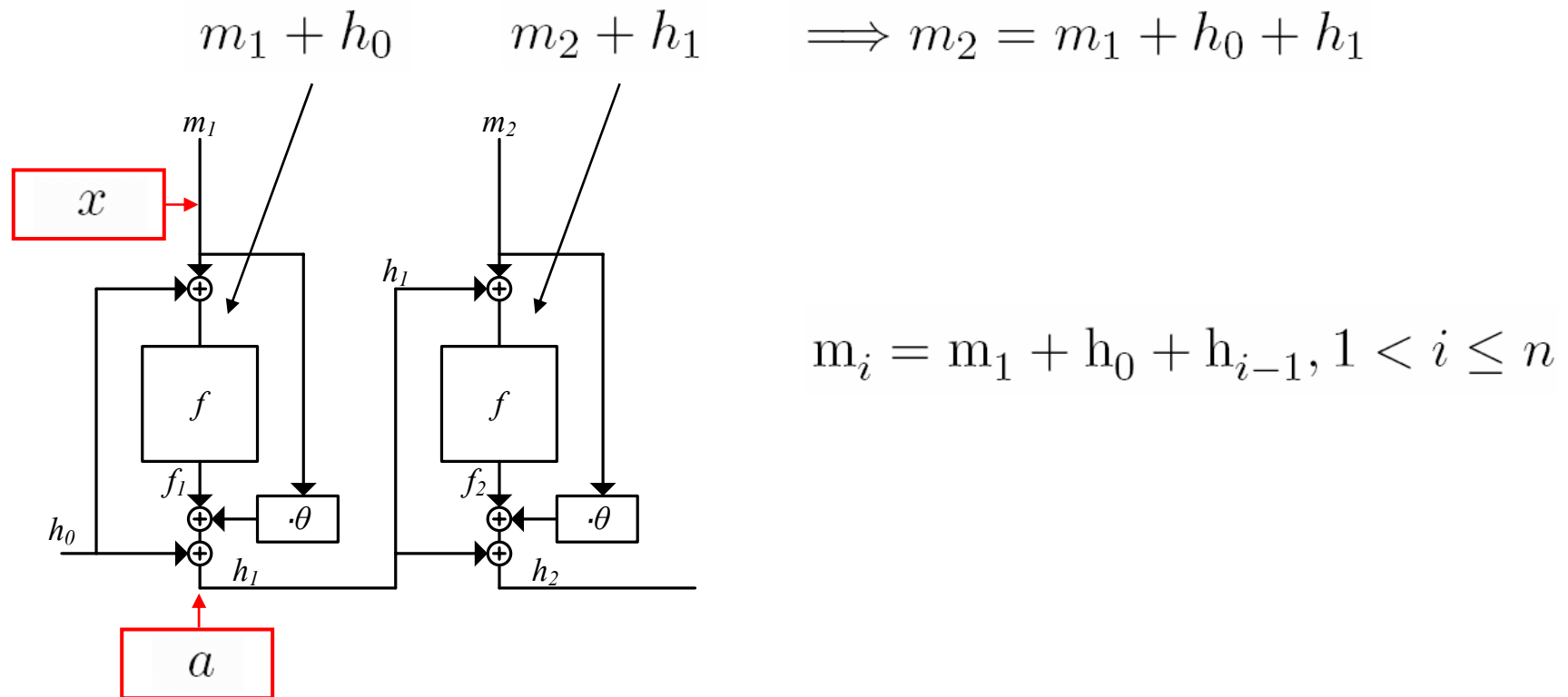
- Introduce non-zero difference  $x$  in  $i = 1, 241, 254, 255, 257$ 
  - 257 message blocks needed
  - 4 message blocks determined by attack
  - 253 message blocks can be chosen arbitrarily



=> collision after iteration 257

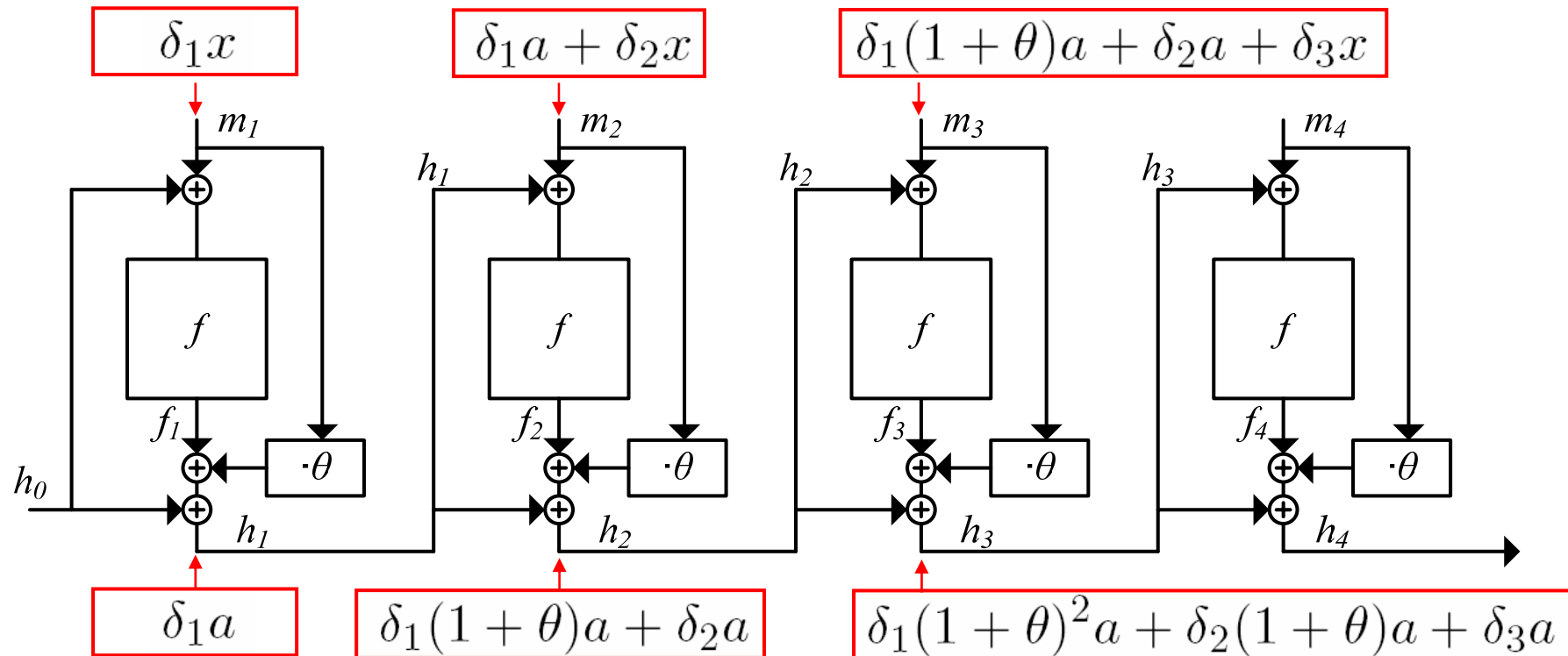
# Second Preimages for SMASH

- Message  $m$ : allow PCP in each iteration



# Second Preimages for SMASH

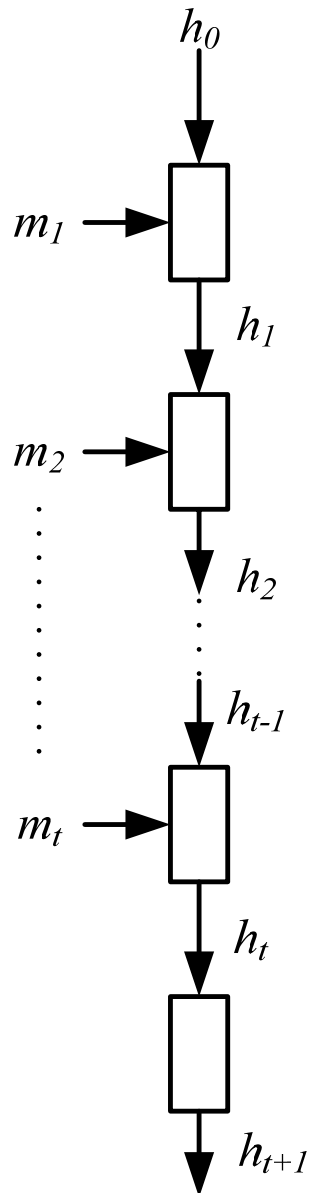
- Message  $m^*(\delta)$  :



$$\delta_i \in \{0, 1\}$$

$$h_n + h_n^* = a \sum_{j=1}^n \delta_j (1 + \theta)^{n-j}$$

## Second Preimages for SMASH



- Given  $m$ , difference  $x$ , and  $\delta$

$$h_n + h_n^* = a \sum_{j=1}^n \delta_j (1 + \theta)^{n-j}$$

- Given  $m$ , difference  $x$ , and  $h_n + h_n^*$ 
  - Set of  $n$  linear equations in unknowns  $\delta_i$

$$A_{n \times n} \times \delta \neq 0$$

- SMASH-256/512:  $A_{n \times n}$  full rank  $\Rightarrow$  solution

## Second Preimages for SMASH

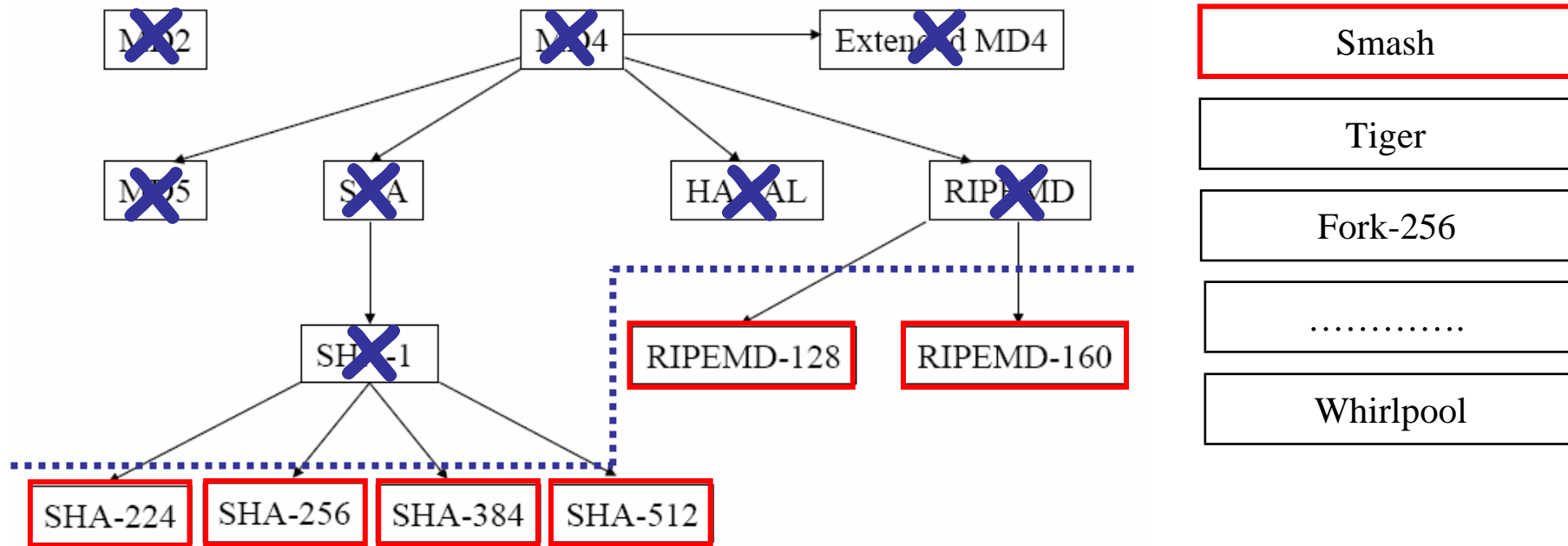
- For  $t$ -block messages
  - $t \geq n$  : same approach applies
  - $t < n$  : same approach but probabilistic
  
- Summary of second preimage attacks

type	message length $t$	number of blocks the attacker can choose	probability
meet-in-the-middle [5]	$\geq 2$	$t - 2$	$2^{-n/2}$
this paper	$\geq n + 1$	$t - n$	1
this paper	$< n + 1$	1	$2^{t-1-n}$

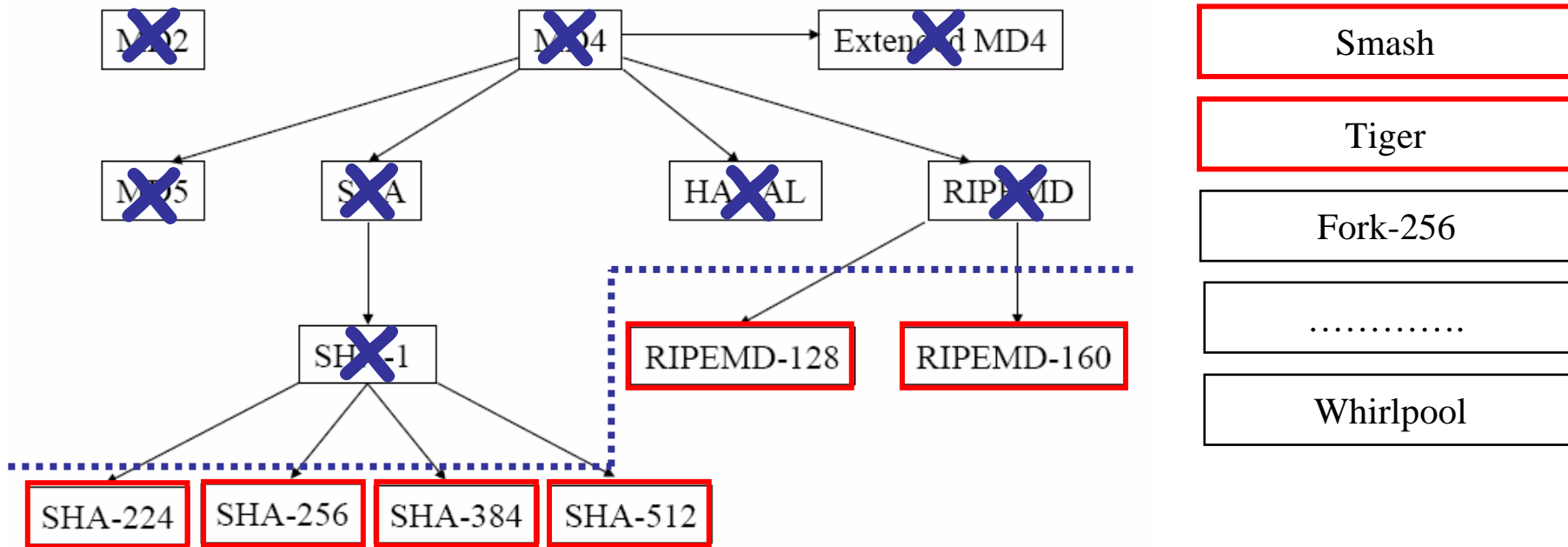
## Summary and Further Work

- Structural analysis of SMASH
  - Second preimages
    - direct construction
  - Special case: collisions
  
- Further work
  - Other hash functions
  - Preimages for SMASH
  - Generalizing strategy
    - FPP and PCP
    - Looking at different compression functions

# Motivation



# Motivation





# Update on Tiger

Florian Mendel, Vincent Rijmen,  
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Hiroataka Yoshida, and Dai Watanabe  
Systems Development Laboratory, Hitachi, Ltd.

Bart Preneel

Katholieke Universiteit Leuven, Dept. ESAT/SCD-COSIC

**presented at Indocrypt 2006**

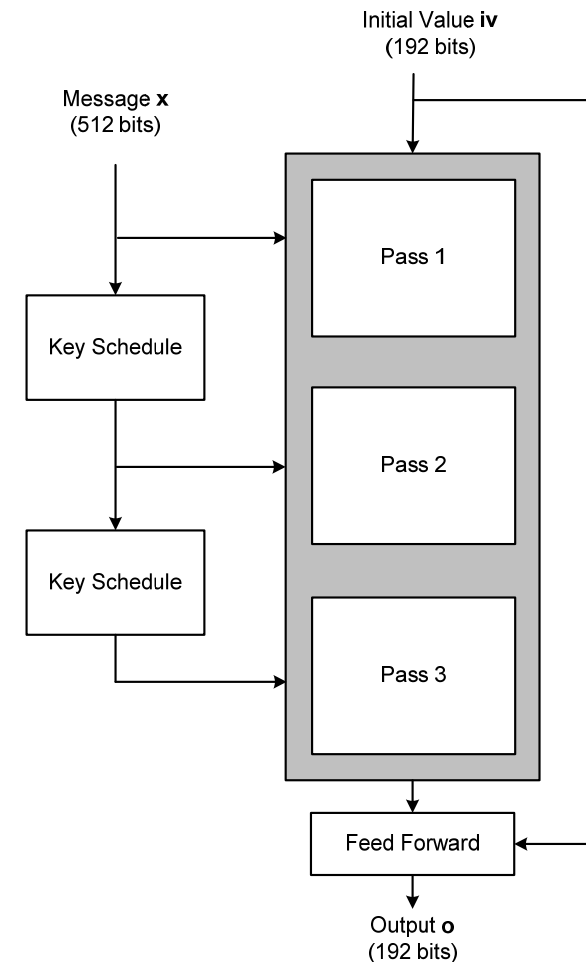
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Graz University of Technology***



# The Tiger Hash Function

- Iterated Hash Function processes 512-bit blocks and produces a 192-bit hash value
- Message expansion
  - 8 64-bit words to 24 64-bit words
- State Update Transformation
  - 3 passes each consists of 8 rounds



## Message Expansion

- The message expansion of Tiger consists of 2 applications of the Key Schedule:

$$(X_8, \dots, X_{15}) = \text{KeySchedule}(X_0, \dots, X_7)$$

$$(X_{16}, \dots, X_{23}) = \text{KeySchedule}(X_8, \dots, X_{15})$$

# Key Schedule

The Key Schedule of Tiger consists of 2 steps

first step

$$Y_0 = Y_0 - (Y_7 \oplus \text{A5A5A5A5A5A5A5A5})$$

$$Y_1 = Y_1 \oplus Y_0$$

$$Y_2 = Y_2 + Y_1$$

$$Y_3 = Y_3 - (Y_2 \oplus ((\neg Y_1) \lll 19))$$

$$Y_4 = Y_4 \oplus Y_3$$

$$Y_5 = Y_5 + Y_4$$

$$Y_6 = Y_6 - (Y_5 \oplus ((\neg Y_4) \ggg 23))$$

$$Y_7 = Y_7 \oplus Y_6$$

second step

$$Y_0 = Y_0 + Y_7$$

$$Y_1 = Y_1 - (Y_0 \oplus ((\neg Y_7) \lll 19))$$

$$Y_2 = Y_2 \oplus Y_1$$

$$Y_3 = Y_3 + Y_2$$

$$Y_4 = Y_4 - (Y_3 \oplus ((\neg Y_2) \ggg 23))$$

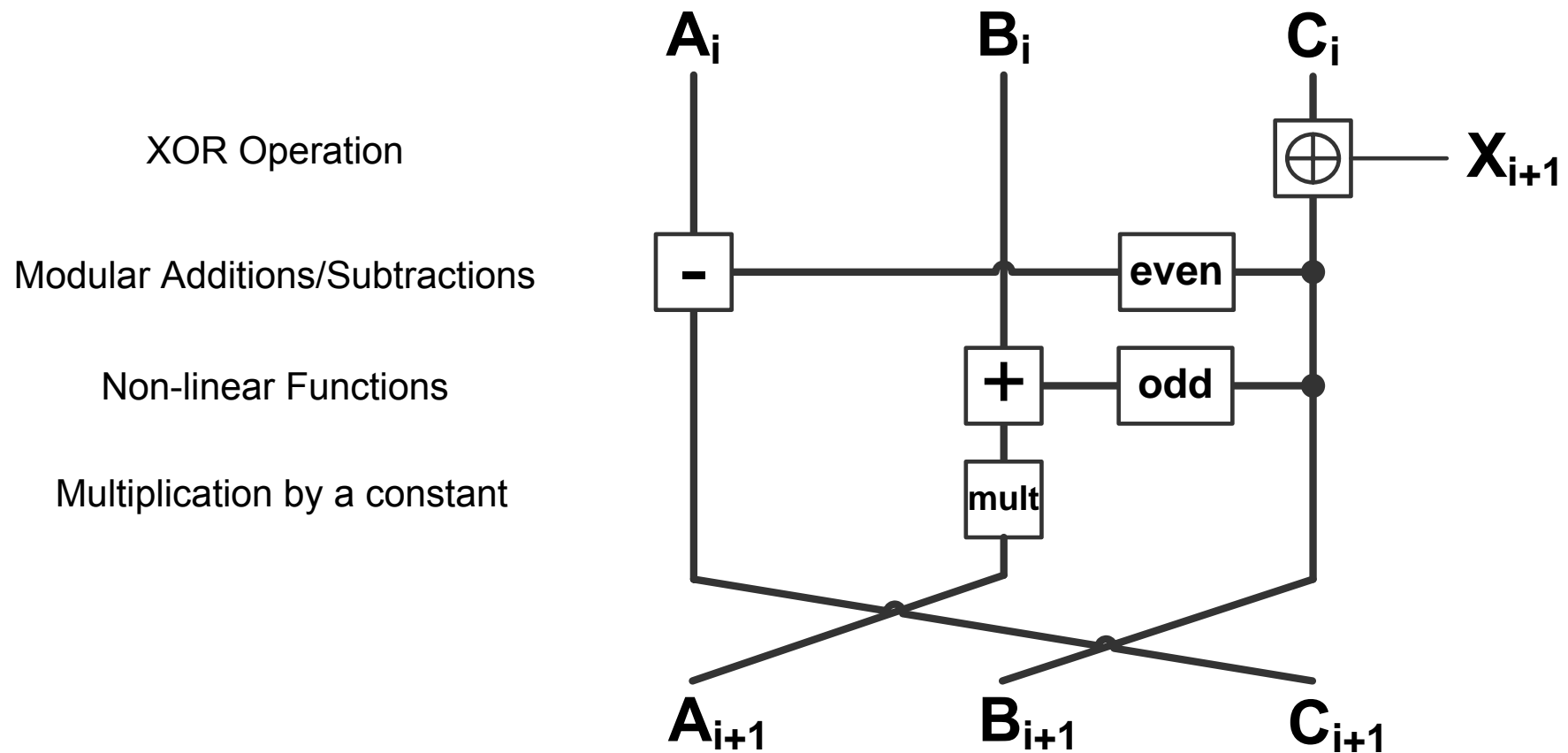
$$Y_5 = Y_5 \oplus Y_4$$

$$Y_6 = Y_6 + Y_5$$

$$Y_7 = Y_7 - (Y_6 \oplus \text{0123456789ABCDEF})$$

# State Update Transformation

- 3 Passes (8 rounds each)



## State Update Transformation

- The non-linear functions *even* and *odd* used in each round are defined as follows:

$$\text{even}(C) = T_1[c_0] \oplus T_2[c_2] \oplus T_3[c_4] \oplus T_4[c_6]$$

$$\text{odd}(C) = T_4[c_1] \oplus T_3[c_3] \oplus T_2[c_5] \oplus T_1[c_7]$$

- 4 S-boxes are used  $T_1, \dots, T_4 : \{0, 1\}^8 \rightarrow \{0, 1\}^{64}$
- At the end of each round  $B$  is multiplied by a constant  $\text{mult} \in \{5, 7, 9\}$ . This constant is different for each pass of Tiger.

## Basic Attack Strategy

- Choose a characteristic for the Key Schedule of Tiger that holds with high probability (ideally with probability 1).
- Use a kind of message modification technique to construct certain differences in the chaining variables, which can then be canceled by the differences in the message words in the following rounds.

## Attack on 16 Rounds of Tiger

- Key Schedule difference for collision in Tiger-16

$$(I, I, I, I, 0, 0, 0, 0) \rightarrow (I, I, 0, 0, 0, 0, 0, 0)$$

- To have a collision after 16 rounds the following difference is needed in the chaining variables in round 7

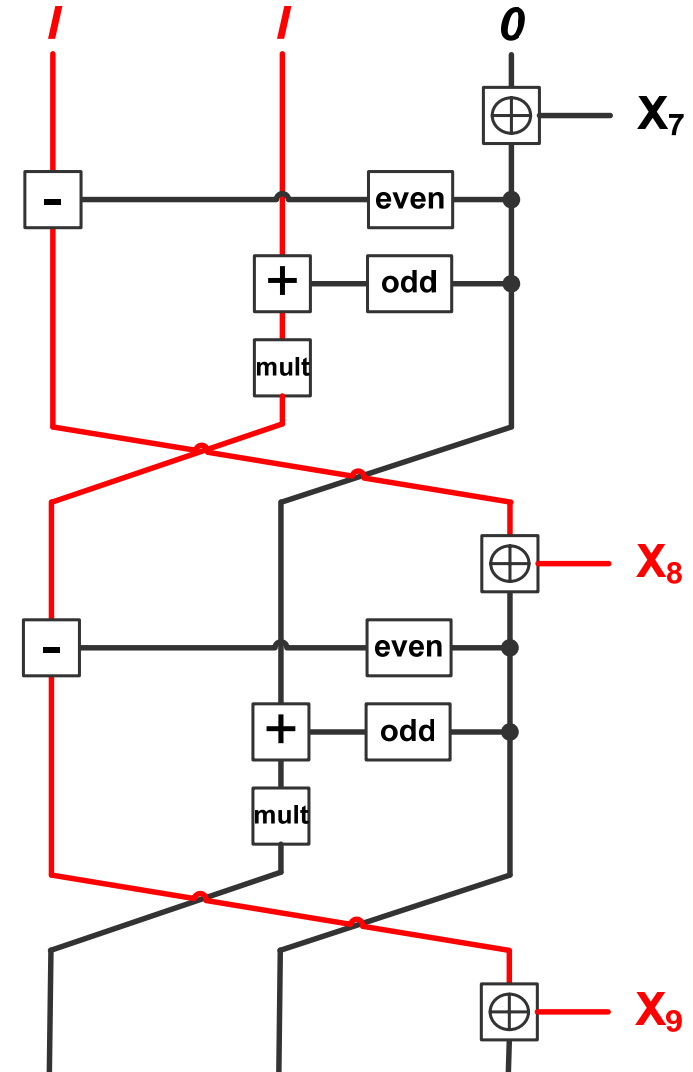
$$\Delta^+(A_6) = I, \quad \Delta^+(B_6) = I, \quad \Delta^+(C_6) = 0$$

- In the attack Kelsey and Lucks use a kind of Message modification technique developed for Tiger to construct the needed differences.



# Collision for 16 rounds of Tiger

- Needed target difference  $(1,1,0)$
- Canceled by words 8 and 9
- Collision after 10 rounds of Tiger
- No difference in remaining words  
=> Collision for 16 rounds

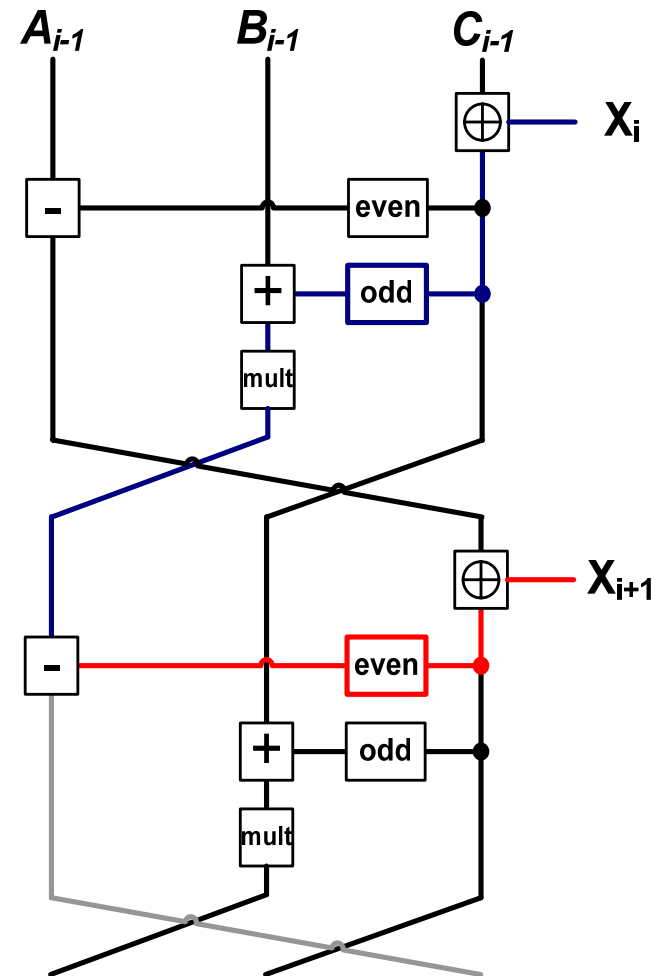


# Message Modification by Meet-in-the-Middle

$$\text{mult} \times (\Delta^+(B_{i-1}) + \Delta^+(\text{odd}(B_i)))$$

$$\Delta^+(\text{even}(B_{i+1}))$$

$$= \delta^*$$



## Message Modification by Meet-in-the-Middle

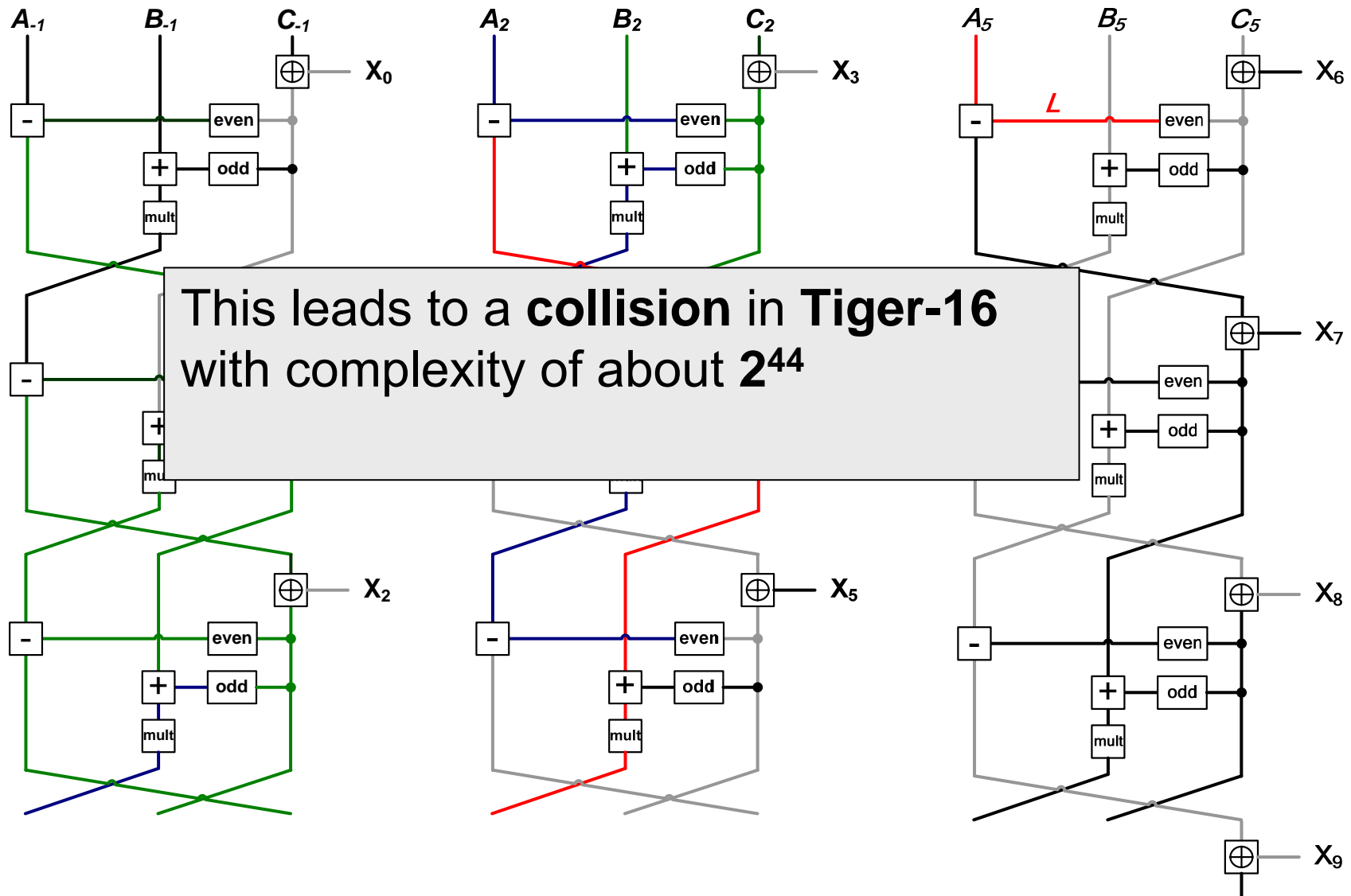
- Use a MITM approach to solve the equation:

$$\underbrace{\text{mult} \times (\Delta^+(B_{i-1}) + \Delta^+(\text{odd}(B_i)))}_{\mathbf{E}} - \underbrace{\Delta^+(\text{even}(B_{i+1}))}_{\mathbf{F}} = \delta^*$$

- Store the  $2^{32}$  candidates for **E** in a table
- For all  $2^{32}$  candidates for **F** test if some **E** exists with  $\mathbf{E} - \mathbf{F} = \delta^*$

This technique takes about  $2^{29}$  evaluations of the compression function of Tiger

# Outline of the Attack



## Going beyond 16 Rounds

- Attack of Kelsey and Lucks (FSE 2006)
  - Collision 16 rounds of Tiger with complexity of about  $2^{44}$
  - Pseudo-near-collision for 20 rounds of Tiger (4 - 24) with complexity of about  $2^{48}$
  
- Extended Attack of Mendel *et al.* (Indocrypt 2006)
  - Collision for 19 rounds of Tiger with complexity of about  $2^{62}$
  - Pseudo-near-collision for 22 rounds of Tiger (1 - 22) with complexity of about  $2^{44}$
  - ...

## A Collision for Tiger-19

- In the attack we use the Key Schedule difference:

$$(0, 0, 0, I, I, I, I, 0) \rightarrow (0, 0, 0, I, I, 0, 0, 0) \rightarrow (0, 0, 0, \cancel{I}, \cancel{I}, \cancel{I}, \cancel{I}, \cancel{I})$$

## A Collision for Tiger-19

- In the attack we use the Key Schedule difference:

$$(0, 0, 0, I, I, I, I, 0) \rightarrow (0, 0, 0, I, I, 0, 0, 0) \rightarrow (0, 0, 0, \cancel{I, I, I, I, I})$$

- Note that the Key Schedule difference from round 3 to 18 is the 16-round difference used in the attack on Tiger-16

$$(I, I, I, I, 0, 0, 0, 0) \rightarrow (I, I, 0, 0, 0, 0, 0, 0)$$

## Outline of the Attack

- Choose arbitrary values for  $A_2, B_2, C_2$  in round 3



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- Choose arbitrary values for  $A_2, B_2, C_2$  in round 3
- Employ the attack on 16 rounds, to find message words  $X_3, \dots, X_7$  and  $X_8[\text{even}], X_9[\text{even}]$  such that the outputs collide after 19 rounds

## Outline of the Attack

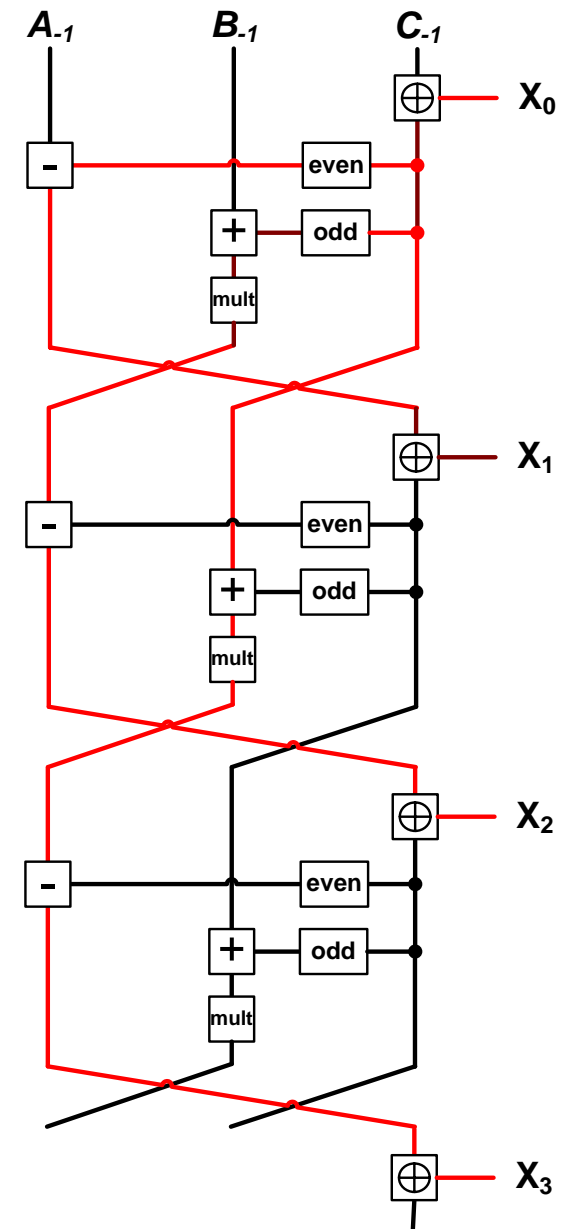
- Choose arbitrary values for  $A_2, B_2, C_2$  in round 3
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- Compute the message words  $X_0, X_1, X_2$  such that  $X_8[\text{even}], X_9[\text{even}]$  are correct after computing the Key Schedule

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- Compute the message words  $X_0, X_1, X_2$  such that  $X_8[\text{even}], X_9[\text{even}]$  are correct after computing the Key Schedule
- Run the rounds 2,1 and 0 backward to get the initial values  $A_{-1}, B_{-1}$  and  $C_{-1}$

# Collision in Tiger-19

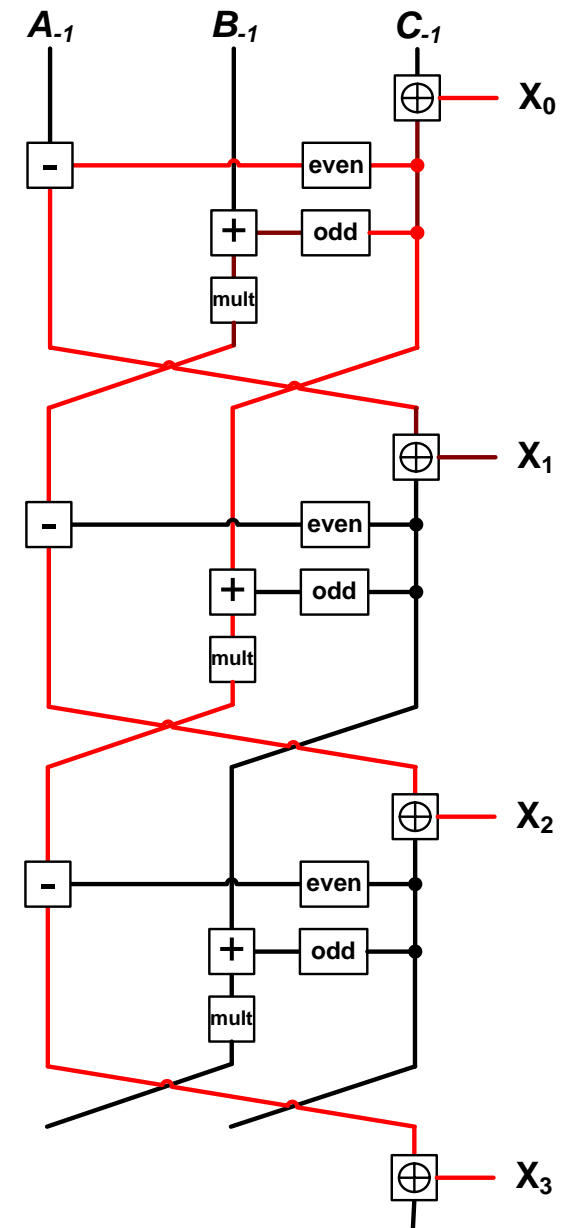
- Use the degree of freedom we have in the choice of the message words  $X_0, X_1, X_2, X_3$  to guarantee that the message words  $X_8[\text{even}], X_9[\text{even}]$  are correct after computing the Key Schedule of Tiger



## Collision in Tiger-19

- Use the degree of freedom we have in the choice of the message words  $X_0, X_1, X_2, X_3$  to guarantee that the message words  $X_8[\text{even}], X_9[\text{even}]$  are correct after computing the Key Schedule of Tiger

This leads to a **collision** in **Tiger-19** with complexity of about  $2^{62}$



# Summary

rounds	type	complexity	$\Delta \rightarrow \Delta$
Tiger-16	collision	$2^{44}$	
Tiger-19	collision	$2^{62}$	
Tiger-19	pseudo-collision	$2^{44}$	
Tiger-21	pseudo-near-collision	$2^{44}$	$(I, 0, 0) \rightarrow (I, 0, 0)$
Tiger-22	pseudo-near-collision	$2^{44}$	$(0, I, 0) \rightarrow (0, I, 0)$

## Summary and Future Work

- Extending the method to find a collision in full Tiger hash function seems to be difficult
  
- By using a weaker attack scenario (pseudo-collisions, pseudo-near-collisions, etc.) it seems to be more likely that the attacks can be extended to full Tiger
  
- Future Work
  - Consider also characteristics for the Key Schedule with lower probability (not only probability 1)
  - Use of non-linear characteristics in the KS of Tiger

# Conclusion

- Recent results in cryptanalysis show weaknesses in many commonly used hash functions
  - MD4, MD5, RIPEMD
  - SHA-1
  - ...
  
- Hash functions that appear to be immune against existing attacks
  - SHA-2 family, RIPEMD-160
    - based on MD4 (!)
  - Whirlpool



Thank you for your Attention

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