Introduction to Cryptographic Hash Functions

Vincent Rijmen

http://www.iaik.tugraz.at/aboutus/people/rijmen/index.php

Hash&Stream, Salzburg, 2007/02/01

Institute for Applied Information Processing and Communications (IAIK) - Krypto Group

Faculty of Computer Science
Graz University of Technology

Overview

- Requirements
- Applications
- Constructions

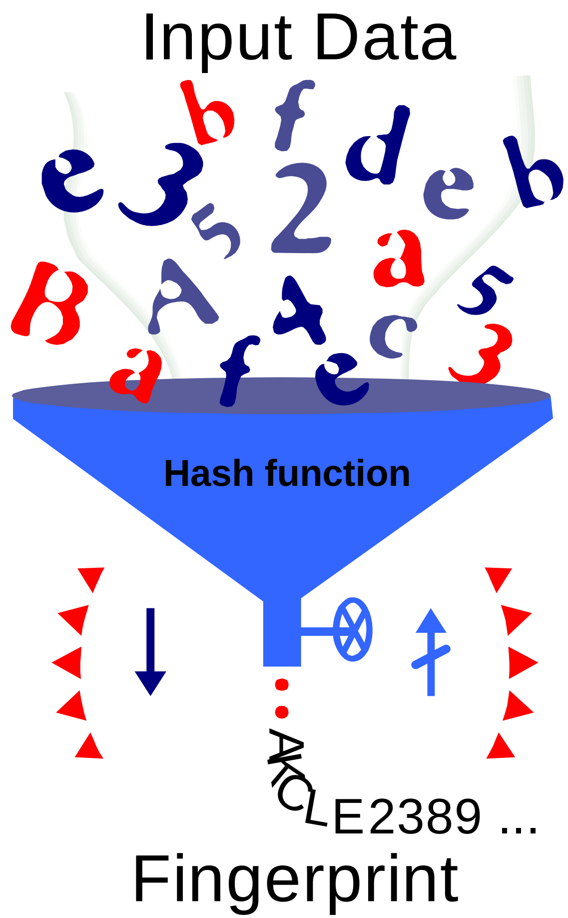
The working principle of a hash function.

Input: data of variable length

Output: fixed-length fingerprint

Fast processing algorithm

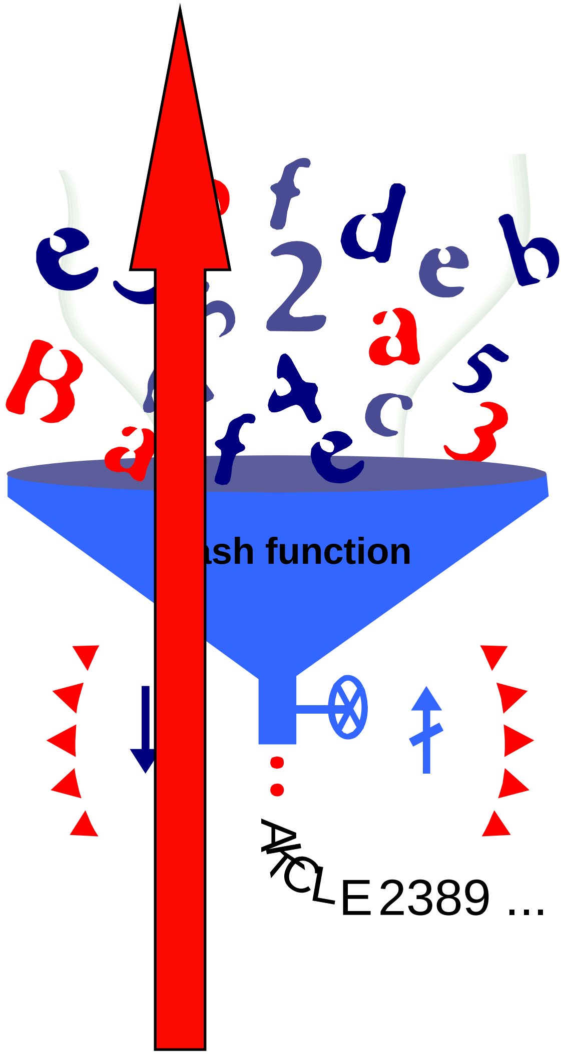
Secure



Pre-image resistance

Given a fingerprint,

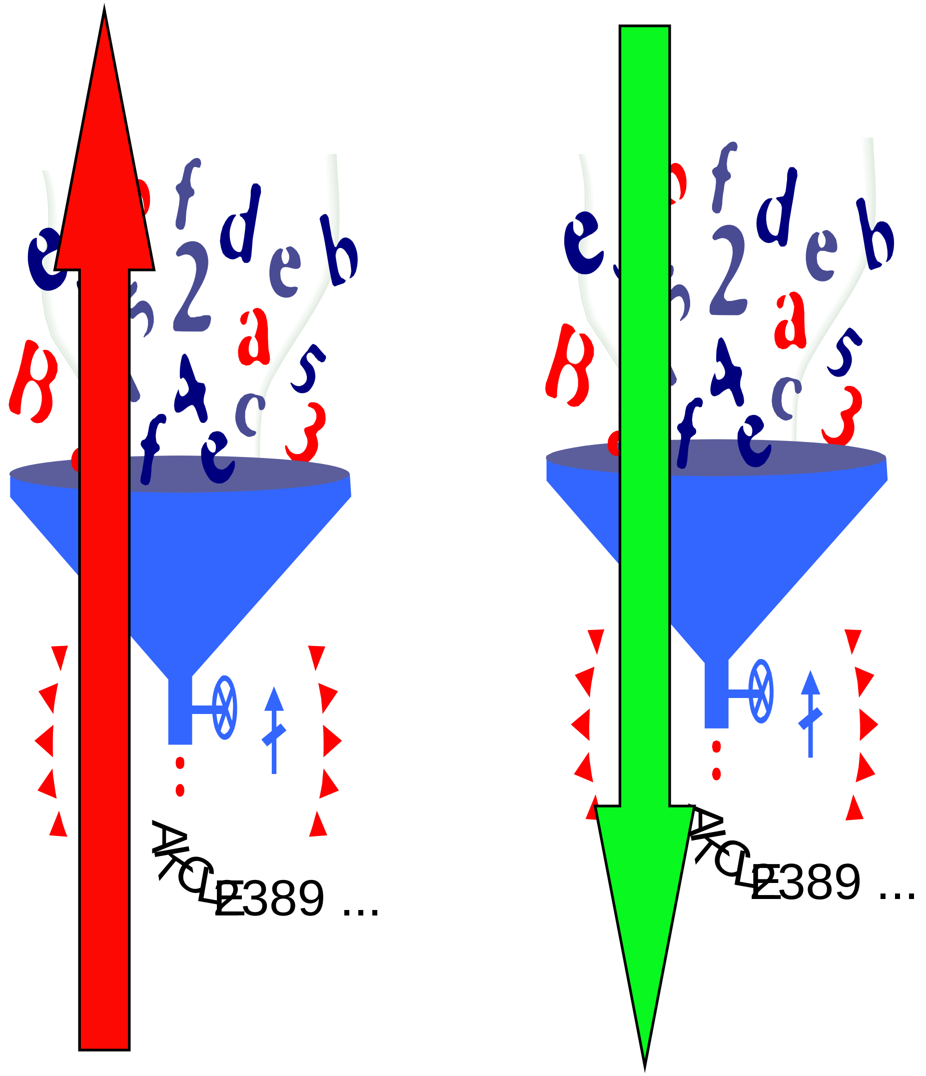
Can't find a corresponding input



Second pre-image resistance

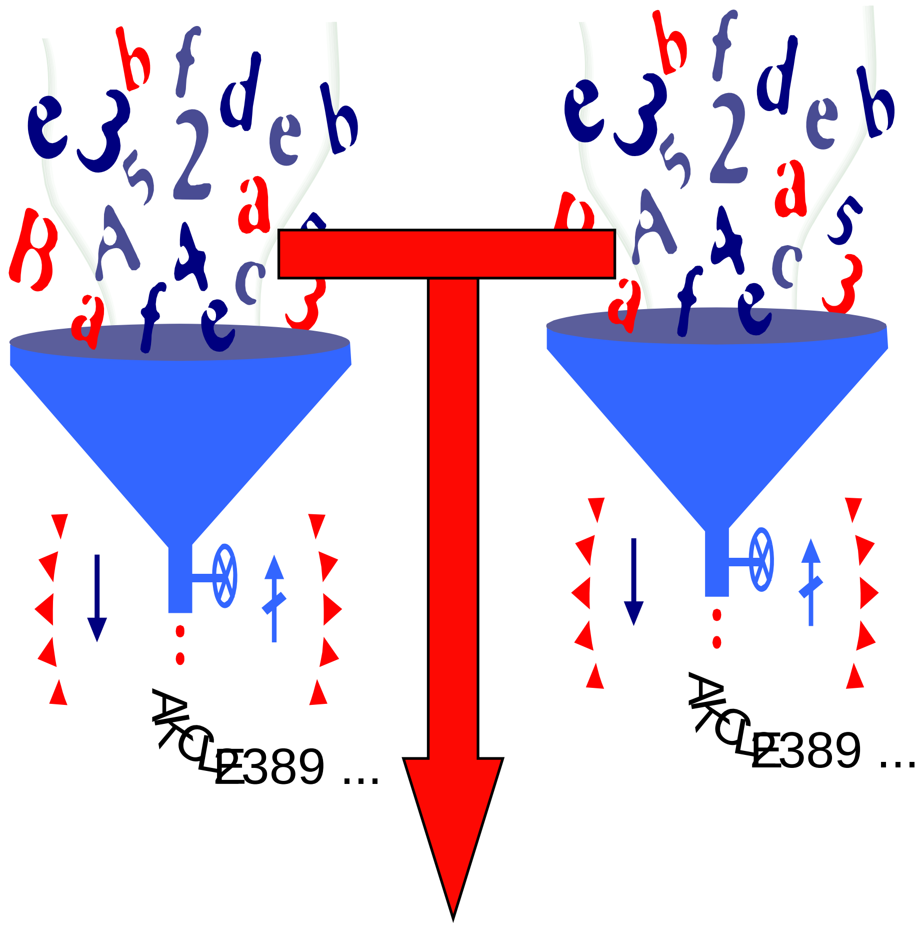
Given an input,

Can’t find a second input with same fingerprint



Collision resistance

Can't find two inputs with same fingerprint

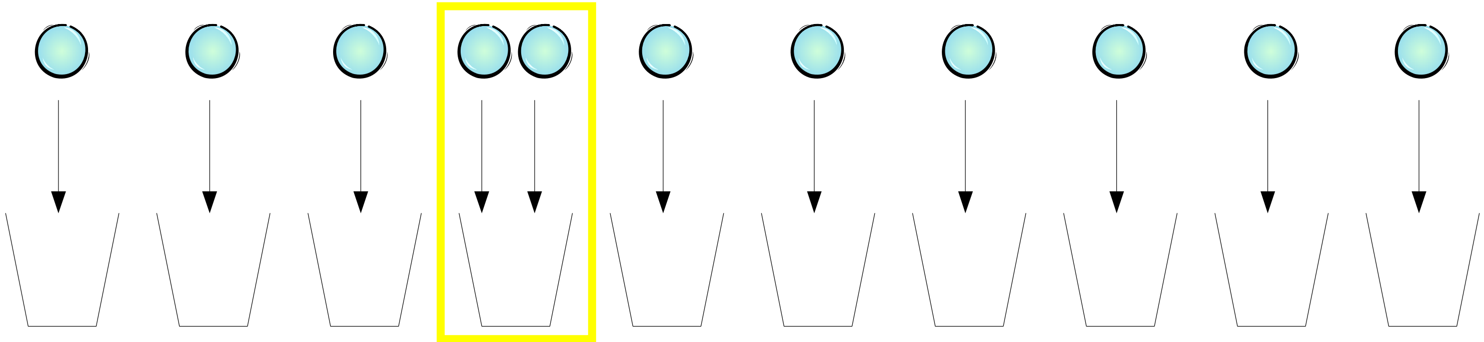


What does collision-resistance mean?

Model:

- Hash function inputs: balls
- Hash function output space: set of bins
- Hash function: throws balls in bins

Pigeonhole principle

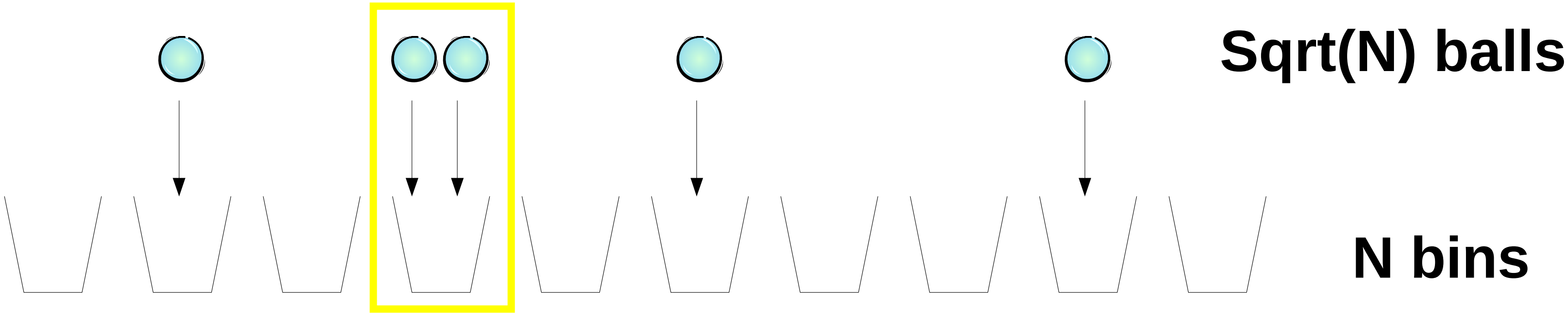


N+1 balls

N bins

If there are more balls than bins, then at least one bin gets at least two balls.

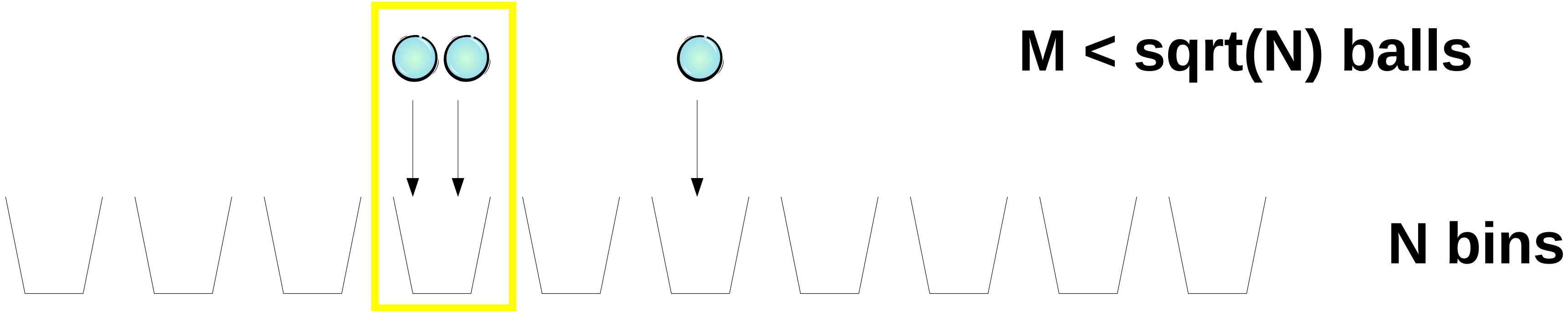
Birthday paradox



More than sqrt(N) balls

⇒ more than 50% chance that two balls go into the same bin.

A collision attack



Less than sqrt(N) balls, but still two balls into the same bin (*academic attack*)

Feasible number M balls (*practical attack*)

Summary

Collision, 2nd preimage, preimage resistance:

- *Computational* properties
- Attack: exploiting internal structure to be more efficient that the generic attack
- Collision attack is easier than 2nd preimage attack is easier than preimage attack

Extra: almost-collisions

What if two inputs give *almost* the same output?

- Called: *near-collision*
- Strictly speaking: no attack
- Usually not wanted
  - May be first step towards collision
  - May interact badly with applications

Applications of hash functions

- Payment schemes
- Secure storage of passwords
- Generation of pseudo-random numbers
- Electronic (digital) signatures
- Commitments

Example 1: Coin flip by telephone

Generate random bit *b*

- 0 = heads; 1 = tails

Choice: head or tails

←

Compare b with choice

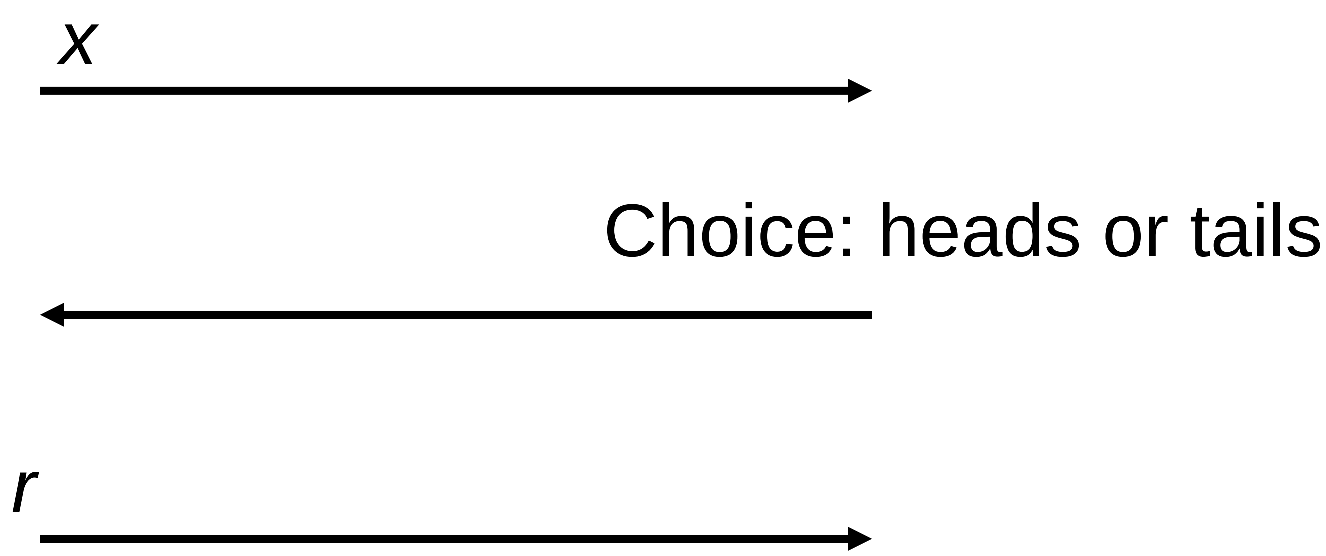
Outcome

→

☹

Commitment

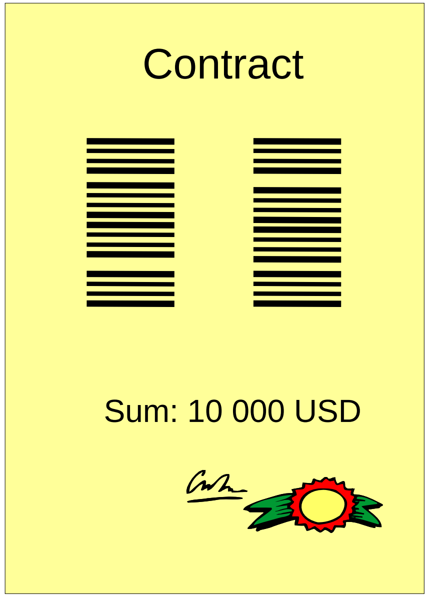
- Generate random number *r*
- Heads/tails = first bit of *r*
- Compute *commitment* *x* = hash(*r*)



Requirements

- Collision resistance
  - Otherwise commitment is not binding
- Preimage resistance
  - Otherwise commitment can be inverted

Example 2: Electronic Signature

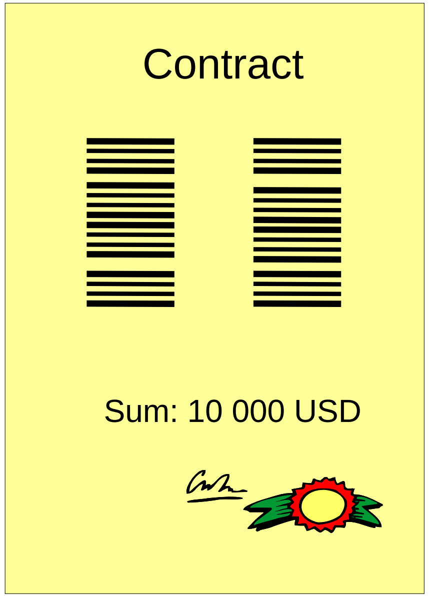


Cut and paste is easy with digital documents

Electronic signatures are *different* for each document

f(content of document, secret of signer)

Example 2: Electronic Signature

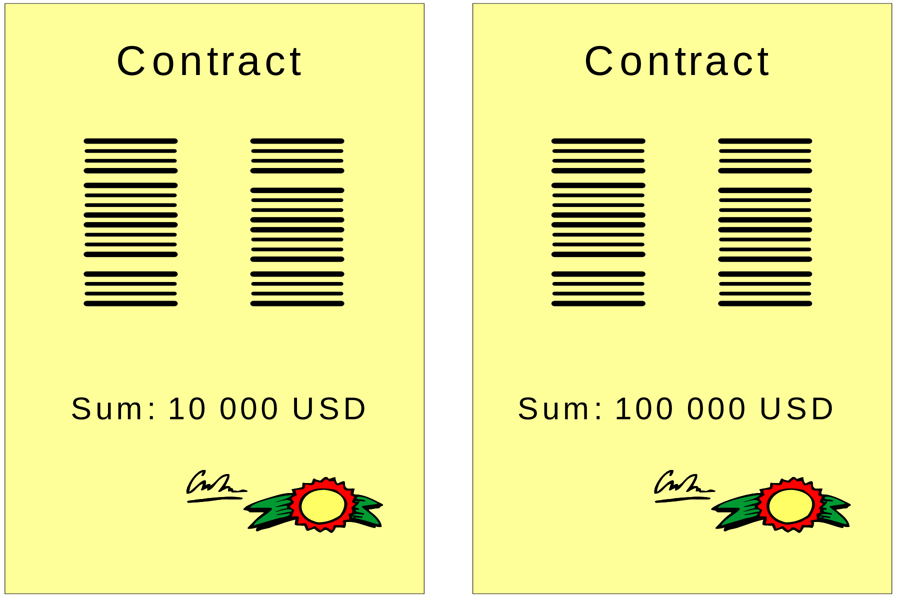


Cut and paste is easy with digital documents

Electronic signatures are *different* for each document

f(fingerprint, secret of signer)

Attack on Electronic Signature



Two documents producing the same fingerprint.

Same fingerprints ⇒ same signatures.

Substitute the original document by a forgery

Constructing hash functions

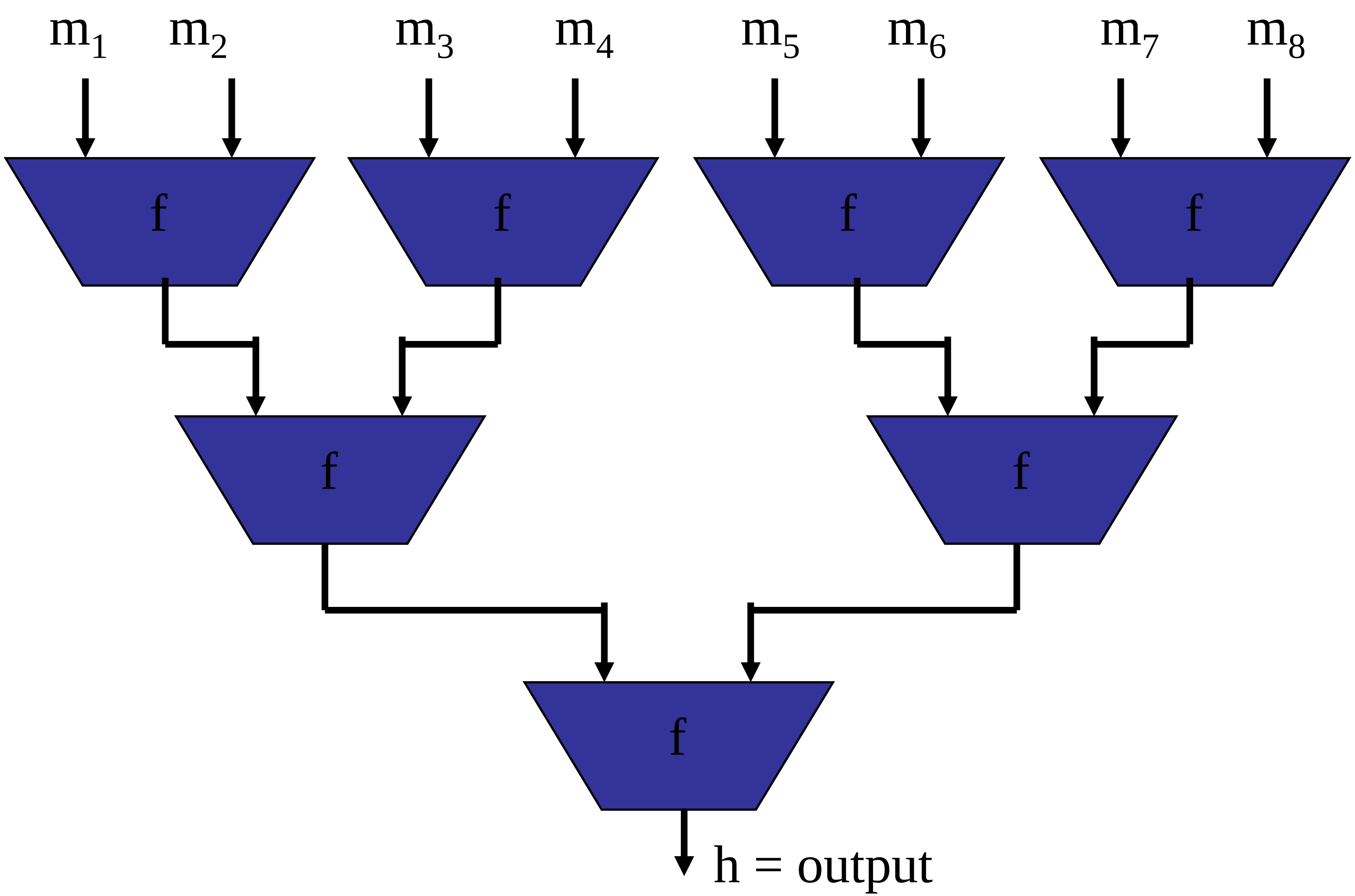
- With or without secret parameter (key)
  - Message Authentication Codes (MAC)
  - Universal hash functions
  - Manipulation Detection Codes (MDC, hash)

- Compression function
- Extension method
  - Tree
  - Merkle’s method

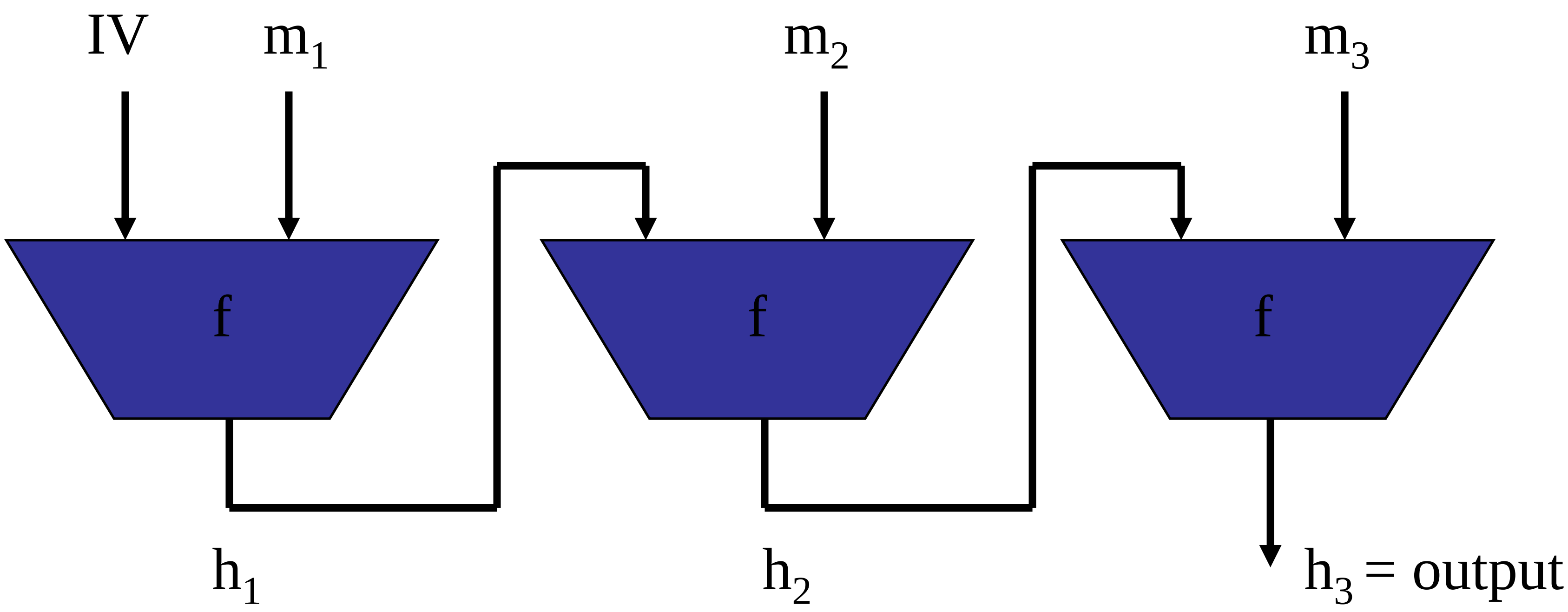
Compression function

- Fixed-size inputs
- Processes a part (*block*) of the message
- Outputs are inputs for next iteration

Tree



Merkle’s method



Merkle-Damgaard Theorem

If:

- Unique padding of message until length is multiple of block length
- Length of (original) message included into padding

Then:

Collision-resistant f $\Rightarrow$ collision-resistant hash

Remarks

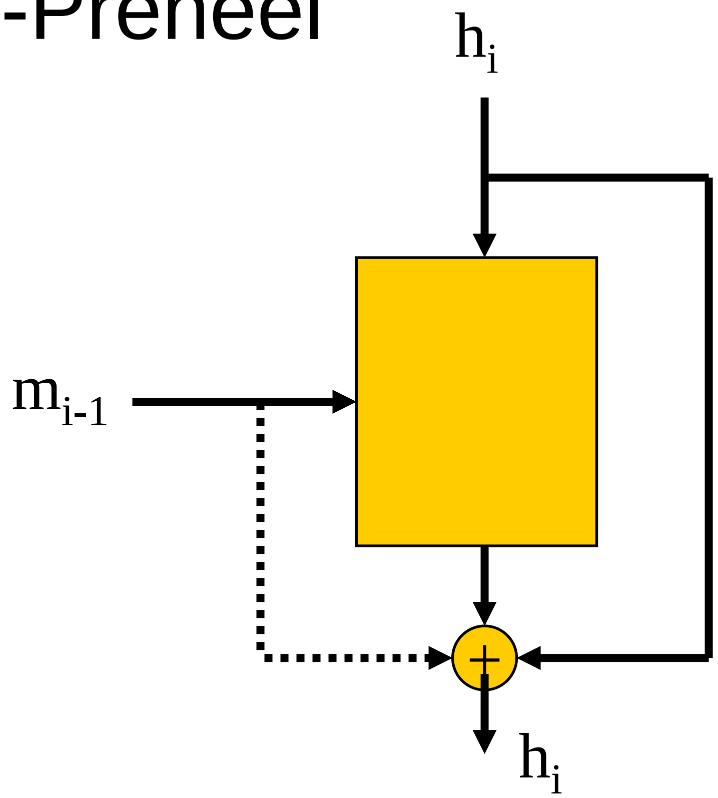
- Padding with length = Merkle-Damgaard strengthening
- No-one uses construction with provably collision-resistant f

Compression function constructions

- Block cipher based
- Dedicated designs
- Modular arithmetic

Block-cipher based designs

- MDC-2, MDC-4 (DES)
- Davies-Meyer, Matyas-Meyer-Oseas, Miyaguchi-Preneel



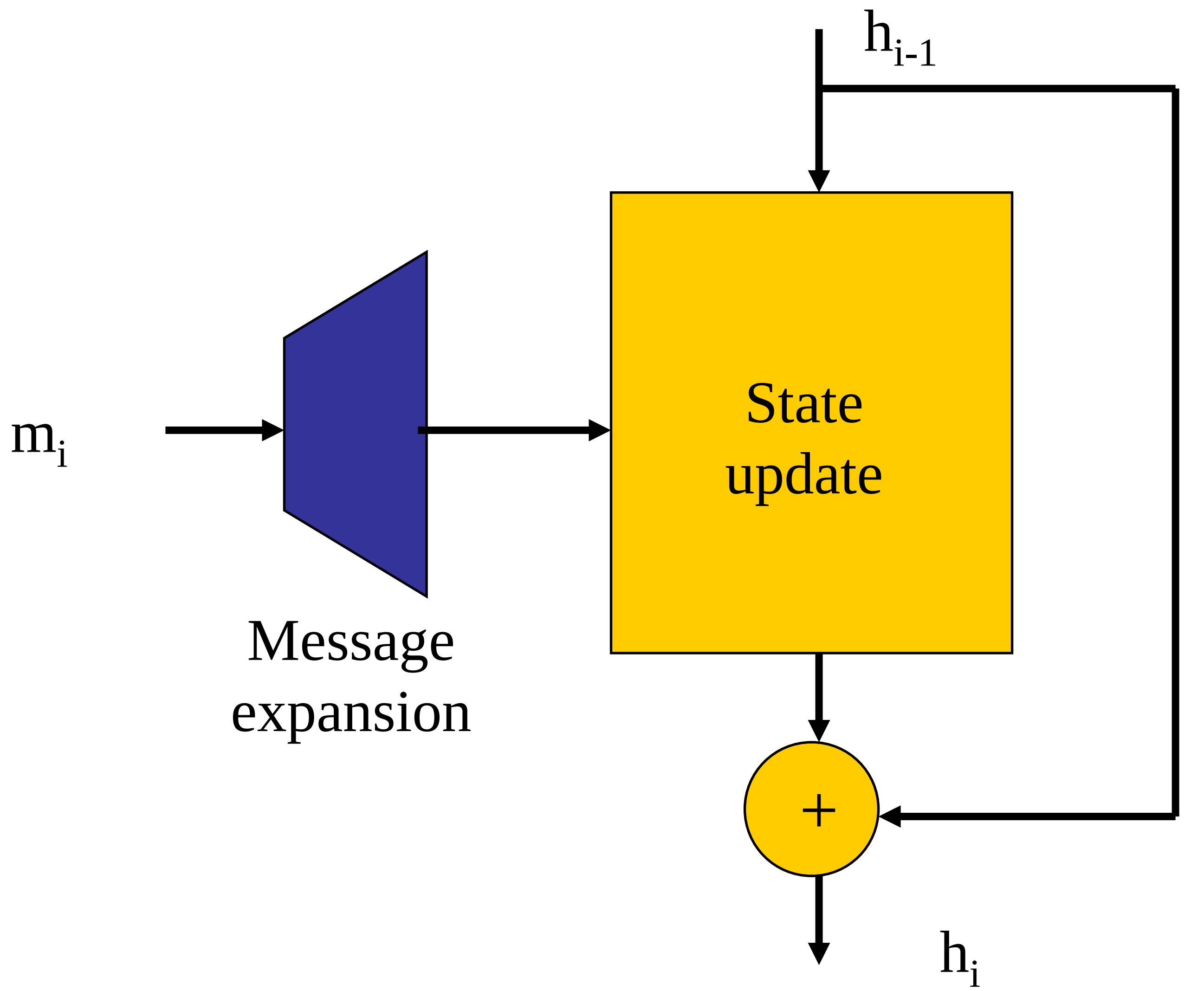
Motivation

- Reduce amount of primitives to be implemented
- Block cipher DES was the only cryptographic primitive with a certification
- Block cipher + feed forward = noninvertible map without apparent structure

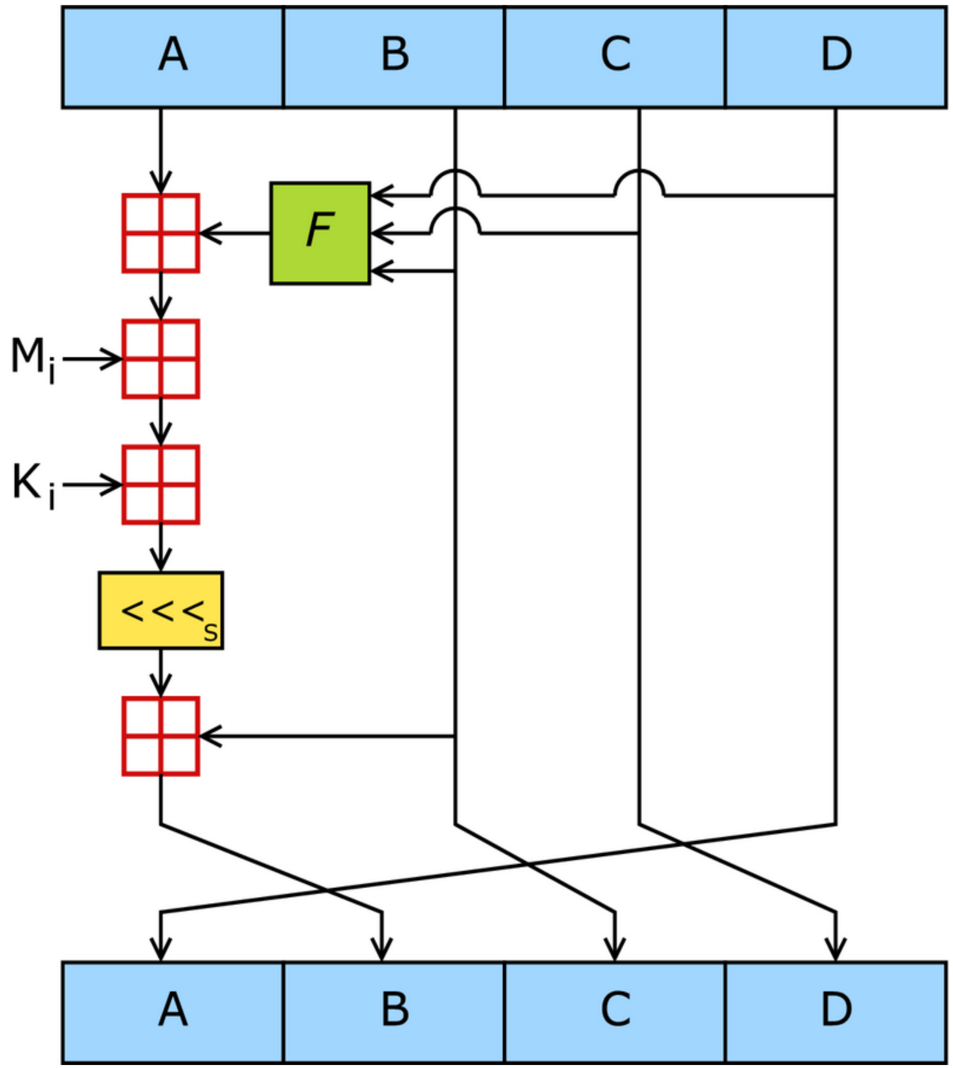
The MD4 family of hash functions

- MD4 [Rivest 1990]
  - Avoid export of encryption software
  - Fast in software on 32-bit CPU’s
- MD5 [Rivest 1992]
  - More secure version
- Extended MD4, RIPE-MD, RIPEMD-160
- SHA, SHA-1, SHA-256, SHA-384, SHA-512, SHA-224
- HAVAL-3, HAVAL-4, HAVAL-5, HAS-1, HAS-2, …

MD5 structure



MD5 step



Picture: Wikipedia

MD5

- Chaining variable: 4x32 bits
- Message input: 512 bits (16x32)
- 4 rounds of 16 steps
- Each step processes one message word
- Rounds differ in
  - Order of message words
  - Shift constants
  - Constants K_i
  - Function F

MD5 functions F

- $IF(x,y,z) = xy \oplus (\sim x)z$
- $IF'(x,y,z) = xz \oplus y(\sim z)$
- $XOR(x,y,z) = x \oplus y \oplus z$
- $F4(x,y,z) = y \oplus (x \text{ OR } (\sim z))$

Other hash functions also use:

- $MAJ(x,y,z) = xy \oplus yz \oplus xz$

Always acting on 32 bits in parallel

Attacks on MD4

- Reduced versions
  - Bosselaers and den Boer (*)
  - Merkle
  - Vaudenay
- Full version, collisions and preimages:
  - Dobbertin (*)

- (*) Also results on extended MD4, MD5
- Not further

The MASH functions

- Based on difficulty of factoring
  - Hence not really symmetric cryptography
  - Slow
- MASH-1(n)

$$y_i = 1111x_{i1}1111x_{i2}1111\ldots x_{it}$$

$$h_{i+1} = (((h_i \oplus y_i) \vee \text{0xF00}\ldots0)^2 \bmod pq) \oplus h_i$$

- MASH-2(n): exponent 2^8+1

Conclusions

- Very little diversity in designs
- Everyone seemed quite confident about the security of SHA-1 and siblings
  - Also reflected in specification of applications